Haley Schilling, a junior philosophy and mathematics major and member of Commonwealth Honors College at the University of Massachusetts Amherst, has been awarded a Beinecke Scholarship. The Fitchburg resident is the first UMass Amherst student to receive the nationally competitive award given to highly motivated college juniors to support graduate study in the arts, humanities, and social sciences. Each Beinecke scholar receives $4,000 immediately and an additional $30,000 while attending graduate school.

One of 20 students named to receive the award this year, Schilling plans to apply to graduate programs in philosophy and eventually become a professor. “Within philosophy, my interests tend to fall under philosophy of science, epistemology, and logic,” she says. “I’m currently compelled by questions that have direct applications to the sciences. What statistical methods should we use? When can we say we’ve found a causal link? Should we prefer simple scientific theories?”

Andrea Nahmod was selected from an exceptional pool of faculty nominations to receive recognition as a 2015-2016 UMass Amherst Spotlight Scholar. This is the most recent in a long series of awards and honors, including the award of a Simons Fellowship, election as a Fellow of the American Mathematical Society, a position as a Sargent-Faull Fellow at the Radcliffe Institute for Advanced Study at Harvard University, and a Simons Research Professorship at MSRI in Berkeley, CA. All these recognize Andrea’s research, which is funded by the National Science Foundation, and her other contributions to mathematics broadly and to our department in particular.

Andrea was selected for her research, for her leadership, and for her scholarly contributions to mathematics in the areas of analysis and partial differential equations. Spotlight Scholars exemplify the outstanding quality and commitment of the UMass Amherst faculty.
NEW CHALLENGE PROBLEMS

Here is a selection of six interesting problems from the 2016 Jacob-Cohen Killam Mathematics Competition.

Problem 1. Which of the real numbers $\sqrt{2016}$ and $\sqrt{2015}$ is larger?

Problem 2. In a square of side-length 1 we are given 9 points. Prove that 3 of the points determine a triangle of area at most $1/8$.

Problem 3. Imagine three equally-charged particles constrained to lie on a circular wire loop which repel each other with a force which decreases monotonically with their distance (in the plane of the circle). Determine the equilibrium configuration – meaning the total force on each particle exerted by other particles is normal to each circle – and show that there is only one such configuration.

Problem 4. Cosmic rays strike the Earth often, some tangentially (angle 0 degrees = 0 radians), some perpendicularly (angle 90 degrees = $\pi/2$ radians), and some at angles in between. Assuming

i) the Earth is perfectly spherical, and

ii) rays are equally likely to come from any direction,

what is the most likely strike angle?
Or equivalently: What is the most likely angle at which a random line in 3-space meets a round 2-sphere?

Problem 5. Three velociraptors are at the vertices of an equilateral triangle and you are at the mid-point of the triangle. One of the raptors is injured and his speed is 1/3 of the healthy raptors’ speed which is 4 times your running speed. Which way should you run to stay alive the longest? Velociraptors pursue the triangle in the direction of the line of sight to the target.

Problem 6. Prove that there does not exist a convex polyhedron such that all of its faces are hexagons. (A polyhedron is a solid in 3 dimensions whose boundary consists of a union of polygons. We say a polyhedron is convex if for any two points of the polyhedron the line segment joining the points is contained in the polyhedron. A face of a convex polyhedron is a polygon contained in the boundary which is equal to the intersection of the polyhedron with a plane.)

Please send your solutions, comments, and feedback to the address on page 19.

DEPARTMENT HEAD’S MESSAGE

Around this time each year, I have the opportunity to reflect over the broader directions of the department, rather than focus on its day-to-day functioning. One major theme is that the number of mathematics majors continues to climb at a fast rate, from 475 two years ago to 583 the following year to 698 this past April, a growth of almost 50% in two years. Students are now able to benefit from participating in three math-major organizations hosted by the department: the Actuarial Club, the Math Club, and the recently installed Student Chapter of the Association for Women in Mathematics. In these pages you can read updates about the second and the third of these, while in last year’s newsletter there was an article about the actuarial concentration. You can also read about some of our most distinguished students who were recognized at our annual awards dinner.

Another area in which the department has grown in the last two years is in the Visiting Assistant Professor (VAP) program. Each VAP is assigned a tenure-track faculty member as mentor and contributes to the research life of the department not only through collaboration with his or her mentor, but also by active participation in seminars, discussions with graduate students and faculty, and other activities. Although they are usually recent PhDs, occasionally VAPs are sufficiently advanced in research to garner their own individual grants from the National Science Foundation, Nathan Totz is one such person. VAPs also make outstanding contributions to the teaching mission of the department. I’m very pleased to report that the number of VAPs has grown from 5 two years ago to 12 next year. Especially given that the university is experiencing some budgetary cutbacks during the current year, the increase in the number of VAPs is a strong indication of the higher administration’s support — thanks in large part to the efforts of our Dean Steve Goodwin — for the value of our VAP program. Indeed, the university is encouraging other departments in the College of Natural Sciences and beyond to adopt our VAP model.

We hope to be able to announce a major upgrade to our VAP program in next year’s newsletter. While we are on the topic of VAPs, I am happy to announce that we had a successful year in hiring them. Joining us will be Jianyu Chen (probability), Jacob Matherne (representation theory), and Dionysios Mantzavinos (applied mathematics). Also, Robin Koytcheff (topology and geometry), who joined us last late August, will continue as a VAP for one more year.

Concerning hiring at the tenure-track level, we succeeded in recruiting a talented analyst, Sohrab Shahshahani. Dr. Shahshahani did his undergraduate work at Wesleyan and at Columbia and his graduate work at the University of Pennsylvania and Ecole Polytechnique Fédérale de Lausanne (EPFL) in Switzerland. He has held postdoctoral positions at EPFL, Mathematical Sciences Research Institute, and the University of Michigan. Shahshahani has diverse expertise in several areas of analysis. He has worked on wave equations and on wave maps in curved backgrounds, a research topic motivated by problems in general relativity, while more recently he has tackled problems in fluid dynamics and Maxwell’s equations.

These pages allow us to highlight only a small fraction of the outstanding work of our faculty in pushing the envelopes of knowledge in mathematics and statistics each year. This year we celebrated Professor Andrea Nahmod for garnering the UMass Spotlight Scholar Award and the MSRI Simons Professorship and Professor Panos Kevrekidis for receiving the Bessel Award. The news that Professor John Staudenmayer was elected as Fellow of the American Statistical Society has just reached us as the newsletter goes to press, and so it will be covered in more detail in the next issue.

This year the torch of Graduate Program Director was passed from Tom Braden to Tom Weston, and Eric Sommers took on the role of Chief Undergraduate Advisor. I’m also grateful to Michael Sullivan and Siman Wong for their excellent work in serving as Associate Heads.

The support of our alumni and friends is invaluable to us. Please send us your news, or better yet, stop in for a visit!
NON-TRANSITIVE SYSTEMS IN GRASSLANDS

1 Introduction and Overview

This year’s Applied Math Master’s Project modeled the non-transitive interactions between three plant species in a prairie and explored the effects that urban development would have on species survival and co-existence. The project was a collaborative effort by students Simon Burhoe, Sandra Castro-Pearson, Emma Dowling, Peng Du, Rachel Gordon, Mark Lowell, Terry Mulllen, Thanyana Sakusiroyngse, Ankita Shankhdhar, and Oliver Spirou under the guidance of Professor Matthew Dobson.

We began by studying non-transitive systems. In a transitive system, if A defeats B and B defeats C then A defeats C. A non-transitive system is one that does not follow these rules. The prototypical example is the game of Rock-Paper-Scissors (RPS), where Paper covers Rock, Rock crushes Scissors, and Scissors cuts Paper. Mathematically, non-transitive systems (often called RPS games) have been examined using discrete and continuous techniques and applied to model multi-player interactions in fields such as biology, economics, and social networking.

We decided to focus on using these systems to study the interactions of the three types of prairie plants. Grasslands all over the world face many threats ranging from agricultural encroachment and introduction of non-native species to ecosystem fragmentation due to construction of roads and other development. In North America, significant portions of grass ecosystems have been destroyed or are currently at high risk. For instance, over 97% of U.S. tallgrass prairies have been lost, and in the Upper Midwest more than twenty plant species are lost, as threats for these species by the U.S. Fish and Wildlife Service. Nonetheless, these ecosystems remain some of the most biodiverse in the world.

In our system, the three players are grasses, forbs, and parasitic plants. Because grasses have the highest birth rate, they outcompete forbs by reproducing faster. Forbs, which are herbs that are not grass or grasslike, have the next highest reproduction rate and are resistant to the latching mechanisms of parasitic plants, and so they outcompete parasitic plants. Parasitic plants, while they have the lowest reproduction rate, can attach themselves to grasses, stealing the nutrients they need from their host and killing the grass. Thus we have the non-transitive system shown in Figure 1.

During the first semester of the project we divided into three groups and explored separate modeling techniques for non-transitive systems. One group of students focused on modeling RPS dynamics using continuous symmetric partial differential equations (PDEs). Another group focused on models based on ordinary differential equations (ODEs). The third group worked on a spatially explicit, stochastic, discrete model which we called the lattice model.

In the second semester, we decided not to pursue the ODE model further because the ODE is not spatial and thus it is necessarily a well-mixed system, and three-player RPS interactions in well-mixed systems lead to the dying off of at least one species. Instead, we adapted the PDE and lattice models so that they would exhibit the specific characteristics of our prairie system. The lattice group also introduced fragmentation to the system caused by roads and other human encroachment and evaluated the effects on coexistence and survival. In particular, they tested spatial geometries of development to determine if some are less harmful than others.

2 The PDE Model

During the fall semester, the PDE team reproduced results from a review paper that uses a continuous symmetric system of PDEs to model the dynamics of three species, grasses, forbs, and bacteria. The PDE was analyzed using von Neumann boundary conditions and the Crank-Nicholson method to discretize the model space (\(\mathbb{R}^2\)). The PDE model has the following mathematical parameters: width of the model space (\(W\)), running rates of grass, forbs, and parasitic plants (\(G\), \(F\), \(P\)), and base death rate (\(\delta\)), impact of parasites on grass death rates (\(\delta\)), season length (\(T_s\)), seed dispersal radius (\(\epsilon\)), number of seeds generated per plant (\(n_p\)), and time step length (\(\Delta t\)).

During summer, which corresponds to a large value of \(T_s\), every plant "runs" at rate \(\beta\). Running means creating a new plant by running along the surface of the ground, plants run only to horizontally or vertically adjacent, empty microsites. The time until the plant attempts to generate an offset is exponentially distributed with parameter \(\beta\). Forbs and parasitic plants that die at rate \(\delta\), that is, the time until they die of random causes is exponentially distributed with parameter \(\delta\). Grasses die at rate \(\delta + n_p\), where \(n_p\) is the number of parasites adjacent to the grass plant.

To simulate this model, we divide summer into \(T_s\) time steps. In each time step, we select \(W^2\) cells randomly — so

\[
\frac{\partial G}{\partial t} = D_G \frac{\partial^2 G}{\partial x^2} - \alpha G + \beta F - \gamma G - \epsilon \rho_0 G
\]

\[
\frac{\partial F}{\partial t} = D_F \frac{\partial^2 F}{\partial x^2} + \alpha G - \beta F - \gamma F - \epsilon \rho_0 F
\]

\[
\frac{\partial P}{\partial t} = D_P \frac{\partial^2 P}{\partial x^2} + \alpha G - \beta P - \gamma P - \epsilon \rho_0 P
\]

where \(D_G\), \(D_F\), \(D_P\) are the diffusivity coefficients for grasses, forbs, and parasitic plants, respectively, and \(\rho_0\) is the initial density of parasites.
that some cells will be selected multiple times, and some not at all—and we check at each cell to see if an event occurs there. For $\Delta t$ small enough, the probability that an event with rate $\rho$ occurs is approximately $\rho \Delta t$. As $\Delta t \to 0$, this is equivalent to the actual exponential probability distributions, but it is much more computationally tractable. So, whenever we check a site occupied by a plant, there is a $\delta \Delta t$ or $(1 + \eta_j)\delta \Delta t$ probability that the plant dies and a $\beta \delta \Delta t$ probability that the plant generates an offshoot. If it generates an offshoot, we choose a random adjacent microsite; if that microsite is empty, we place a plant of the parent’s species there.

At the end of summer, each plant generates $S$ seeds. For each seed we generate a dispersal distance, uniformly distributed between 0 and $2\pi$, and we allocate the seed to the corresponding cell.

Once we have allocated seeds for every existing cell, we kill all the plants since we are assuming that they are annuals and die over winter. We then germinate the seeds, producing new plants of the same species as the species that generated the seed. If a microsite has seeds from more than one species, then which species occupies the site after winter is determined randomly, with the probability proportional to the number of seeds from each species. So, for example, if two grass seeds, three forb seeds, and four parasite seeds land on the same site, there is a $\frac{2}{3}$ probability a grass plant will sprout, $\frac{3}{8}$ that it will be a forb, and $\frac{7}{12}$ it will be a parasite. We then resume the running simulation. For this base model, there is indefinite coexistence of all three species provided that $W$ is large enough and we choose suitable parameters $\beta, \delta, I$. Figure 4 captures coexistence in one simulation of the lattice model.

When we initialize the lattice model, we also “develop” certain squares, meaning they have been taken over by human activities such as roads, farms, etc. This means eliminating them from participation in the model, they are permanently empty squares that can never be occupied by any species and have no effect on the model except to take up space. We select these developed squares first by randomly picking $n_C$ points to be cores, and we develop them. We also pick two points for each of our $n_L$ lines to be the end points of the lines, and we develop every point between the lines. Cores represent point sources of development, such as cities or farms, while lines represent linear sources of development, such as roads. After we have placed the cores and lines, we randomly select a line or a core, then randomly pick a cell in the selected core or line, then perform a random walk until we reach an undeveloped square, and we change it to developed. We repeat this process until we reach the desired amount of development $N$. A typical example will look something like Figure 5.

With the base development model in place, we ran simulations for different levels of development to see how that affected the long-term probability of species coexistence.

We set a fixed time $T_{\text{max}}$ as a cutoff for the simulation and fixed the width of the lattice $W$ at 500, the seed-dispersal radius $R$ at 3, and the number of seeds $S$ at 2. We ran over 50,000 simulations, taking note of the amount of area developed, the kind of development used, and whether or not a species died off during each simulation. We then used a logistic regression model with a second-degree polynomial to analyze the effects of several parameters on our response variable, the probability of a species dying off within time $T_{\text{max}}$. The predictors were the proportion $r$ of the lattice that was developed and two indicator variables for the type of development: $d_L = 1$ if there is all line development and $d_C = 0$ otherwise, $d_L = 1$ if there is all core development and $d_C = 0$ otherwise. Thus, if $d_L = d_C = 0$, we would have a mixture of cores and lines in the simulation.

A plot of the data before and after the analysis can be seen in Figure 6. The results indicate a surprising phenomenon. Initially increasing development leads to a decrease in the probability of die-off regardless of the type of development implemented. However, as expected, large amounts of development increase the probability of species die-off regardless of development type. We believe that the reason for the initial decline in die-off is that a small to moderate amount of development can help decrease the well-mixed aspect of the system, which in turn allows for survival and coexistence. However, with high development, the amount of space available becomes drastically reduced, making it harder for all three species to sufficiently grow and reproduce. In addition, core development leads to the highest die-off
probability while line development produces the lowest. We believe that line development produces the lowest die-off probability because of the geometric nature of lines; they have a high perimeter-to-area ratio, which leads to more microsites and their neighbors being impacted by the development. This means that more individual plants have an effectively reduced seed-dispersal radius, and the system is less well-mixed as a result.

From the analysis, we conclude that it is not only the amount of development that matters, but also its geometry. For non-transitive systems, the optimal choice is development that reduces the effective well-mixedness of the prairie without completely disconnecting it.

4 Conclusions

We developed PDE and lattice models that captured the asymmetry, non-transitivity, and biodiversity of the prairie GFP system. We were able to verify that both the amount and geometry of development affect species survival. The best geometry decreases well-mixedness in the model without disconnecting it. Surprisingly, small amounts of development showed improved biodiversity while, as expected, large amounts increased the probability of die-off. As future endeavors, we would like to introduce seasonality and human development to the PDE model. For the lattice model, we would like to perform an in-depth sensitivity analysis on parameters such as seed-dispersal radius, number of seeds produced, etc. We would also like to introduce measures for model well-mixedness and alternate measures of biodiversity in order to further analyze the effect of development on the GFP system.

KATSOULAKIS AND COLLEAGUES DEVELOP RELIABLE PREDICTIVE MODELS FOR CATALYSIS AND ENERGY MATERIALS

In the February 2016 issue of Nature Chemistry, Mathematics and Statistics Professor Markos Katsoulakis and colleagues at the University of Delaware introduced a new approach to improve the predictive capabilities of mathematical models of complex chemical-reaction networks arising in materials science. Their mathematical and computational framework provides key steps towards guiding experiments and predicting new materials for catalysis and other energy-related applications such as renewable hydrogen-production for fuel cells.

Mathematical models are a primary tool used both to understand and to predict complex phenomena arising in weather and climate, biology and medicine, traffic patterns, social networks, and other collective systems. Though potentially powerful—it is part due to the availability of inexpensive and powerful computational tools—such models are vulnerable to numerical errors, model uncertainty, or even model misspecification. Furthermore, realistic mathematical models are asked not only to account for increasingly complex systems with a very large number of variables, but also to incorporate available data from different scales. For instance, models in materials science must handle variables and time scales ranging from the atomic level all the way to the everyday macroscopic scale. Many such models involve very complex chemistry and necessarily have a large number of parameters—including reaction constants, activation energies, and external inputs—sometimes in the thousands or even millions, which in turn should be determined from experimental or quantum-simulation data.

The methods of global sensitivity-analysis developed by Katsoulakis and collaborators account in a systematic manner for hidden correlations between model parameters and show how they propagate and affect model predictions. These methods demonstrate that failing to account for correlations and model constraints can lead to wrong predictions, unnecessarily high dimensionality, and intractable statistical inference from data. On the other hand, when such effects are incorporated, the uncertainty-quantification methods developed by these researchers also show that models can have remarkable predictive capabilities at macroscopic scales, despite the significant model-uncertainty and parameter-uncertainty that are inherently present in materials. Another key challenge here, in contrast to “big data,” is that the available data (experimental and computational from electronic-structure calculations) is limited and expensive to obtain, but can provide crucial insights. In particular, adding new data points needs to be done in a principled manner so that the information gain in the prediction is maximized. The developed methods also relate closely to recent mathematical work by Markos Katsoulakis, Luc Rey-Bellet, their students, and their collaborators on information-theoretic methods for uncertainty quantification, which point the way toward a general multi-scale mathematical theory for reliable predictive modeling and computing of complex systems in materials and elsewhere.

Markos adds, “In this publication, we provide the first rationalization and quantification of why kinetic models under-predict experimentally measured reaction rates by a considerable degree. We also show that, despite significant model uncertainty, our correlative uncertainty-quantification methods, combined with experiments, provide key insights into catalyst active sites and missing reaction steps that can be used to improve the predictive ability of the physical model itself. It’s the same with model parameters. When we looked at correlations among parameters and their dependence on each other, we realized that the errors were not nearly as large as we thought they were.”

DOBSON, KATSOULAKIS, AND TEAM OF RESEARCHERS RECEIVE $3.1 MILLION DARPA GRANT

During September 2015 Matthew Dobson and Markos Katsoulakis, along with students, postdocs, and a team of researchers from Brown University, University of Delaware, University of North Carolina-Chapel Hill, and University of California San Diego received a $3.1 million award from U.S. Defense Advanced Research Projects Agency (DARPA). This grant focuses on developing a predictive modeling framework for the reliable computational design of novel, superior, and/or lower cost materials for applications to catalysis, energy production, and energy storage.

To date there is strong experimental evidence that novel material architectures can provide unprecedented performance. Therefore a reliable computational framework for the prediction of such materials can be a critical ingredient for decreasing development cost and time-to-market. In view of our ever-increasing computational capabilities, such a computational framework is already emerging. However, despite their sophistication, computational models are not always predictive. This can be attributed to both model uncertainties as well as the vast spatiotemporal discrepancies between the scales at which we need to make predictions and the scales at which reliable simulations are feasible. In particular, although Uncertainty Quantification (UQ) for this broad class of problems is still at an embryonic stage, nevertheless, because it is intimately linked to reliable computational predictions, UQ is a critical component for guiding the synthesis of materials and experimental assessment of model predictions. The goal of the new grant is to bridge this scientific and ultimately technological gap via the interdisciplinary expertise and collaborations of the participating teams and their researchers. From a mathematical perspective, this research represents a new direction for applied and computational mathematics, bringing together applied probability, uncertainty quantification, high-performance computing, and data science.

The team at UMass Amherst leads the research thrust on developing scalable algorithms and the enabling mathematical methods for uncertainty quantification and predictive computing for multi-scale, multi-physics systems with from hundreds to millions of uncertain parameters, often correlated due to underlying physicochemical principles and constraints. Besides applications to energy and materials, such challenges are typically encountered in many chemical and biological processes, geosciences, and epidemiology as well as in various types of complex networks. Furthermore, the UMass team will develop fast uncertainty-quantification and sensitivity-screening methods for extreme and rare events and will study how the overall behavior of complex, stochastic systems such as network dynamics is ultimately determined by the occurrence of rare events. Additional background on this research project is given at http://www.supercalcingonline.com/latest/58851-katsoulakis-develops-new-framework-for-materials.
INTERNATIONAL SCHOLAR AWARD

Panos Kevrekidis Wins International Scholar Award

During November 2015 Panos Kevrekidis was informed that he would be awarded the Friedrich Wilhelm Bessel Research Award from the Alexander von Humboldt Foundation of Germany for his outstanding research accomplishments. The award ceremony will take place July 6–8, 2016 in Berlin. Panos is one of only 20 researchers worldwide to receive the honor this year. The award, valued at €45,000 or about $49,000, includes the invitation to spend up to a year on long-term research projects with specialist colleagues of Panos’s choice in Germany.

Panos, whose work straddles mathematics and physics, says he hopes to spend a total of perhaps 12 months in three-month segments starting next year with researchers at the University of Heidelberg and the University of Hamburg. There he plans to collaborate with experimental and theoretical groups in physics, especially atomic physics and nonlinear optics, and to use the applied mathematical tools and techniques his group is developing to address relevant physical problems of interest to these groups.

MATHMATICAL BIOLOGY: SOPHISTICATED TOOLS FOR ANALYZING A VARIETY OF BIOLOGICAL MODELS

Editors’ Note: In this article Patrick Flaherty and Yao Li, two newcomers to the department, describe their research in mathematical biology. Their research contributes to the growth of ongoing research in mathematical biology being carried out by other members of our department, both in statistics and in mathematics. Background on Patrick and Yao can be found in the article “New Faces in the Department,” on page 12.

Methods for Analysis of Large-Scale Genomic Data Sets

In 2011, leadership at the National Human Genome Research Institute in the National Institutes of Health published an article in Nature describing their perspective on the future of genomic medicine titled “Charting a course for genomic medicine from base pairs to bedside.” In that article they state, “The major bottleneck in genome sequencing is no longer data generation — the computational challenges around data analysis, display, and integration are now rate-limiting. New approaches and methods are required to meet these challenges.”

In particular, they identify five primary rate-limiting steps for translating bench science to bedside therapeutics: (1) data-analysis, (2) data integration, (3) visualization, (4) computational tools and infrastructure, and (5) training. Evidently, the analysis of large, heterogeneous, genomic data sets has risen as a national scientific priority. In recent years, the National Science Foundation and the National Institutes of Health have come together to support joint initiatives at the intersection of mathematics, statistics, and biology designed to meet these challenges.

Dr. Patrick Flaherty’s research focuses on three of those areas: data analysis, data integration, and training. One of the main goals of his research is to develop rigorous, computationally efficient, statistical methodology for large-scale genomic data analysis. His research focuses primarily on two specific statistical problems: detecting rare mutations in mixed samples and estimating intra-tumor heterogeneity.

Patrick and his group have developed statistical methods to use high-throughput DNA sequencing data to identify rare mutations in blood and other heterogeneous samples. In one study, they identified a point mutation in a human H1N1 influenza sample that confers resistance to a primary antiviral <http://www.ncbi.nlm.nih.gov/pmc/articles/PMC3245950/>. They have since extended their initial statistical model to handle data with more uncertainty <http://bioinformatics.oxfordjournals.org/content/31/17/2785>.

The second problem on which Patrick’s group focuses is intra-tumor heterogeneity. In solid cancers, a tumor is often a heterogeneous mixture of cells. Some of those cells may respond well to a particular chemotherapy, while others may not. The goal is to estimate, for a particular patient, the actual mixture of cancerous cells in the tumor so that individualized combination therapies can be developed. This problem is statistically challenging because the mixture can be complex and highly variable. In recent work, they developed a hierarchical Bayesian statistical model and a companion variational inference algorithm to solve this problem for large-scale genomic data sets <http://bioinformatics.oxfordjournals.org/content/31/22/225>.

Looking forward, Patrick and his group are focused on three problems. First, they are extending this work to larger data sets — a goal that presents both statistical and computational challenges. Second, they are working to translate their methodological work to diagnostic applications in partnership with colleagues in the biotechnology industry and at Massachusetts General Hospital. Finally, they are working to educate the next generation of statisticians to meet the rate-limiting challenges involved in the analysis and integration of biological data.

Methods for Analysis of Complex Biological Networks and Neuron Field Models

Dr. Yao Li’s primary research field lies at the interface of applied dynamical systems and applied probability, with a focus on ergodic properties and limit properties of non-equilibrium systems in high and infinite dimensions. His research in mathematical biology has two directions: the quantification of systematic measures of complex biological networks and the study of non-equilibrium neuron field models. In general, real-world biological systems are inherently heterogeneous, high dimensional, and out of equilibrium. It is important to develop novel and sophisticated analytical and computational tools to respond to these challenges.

In order to characterize certain global properties of a complex biological network, people in systems biology have proposed many systematic measures. Systematic measures including complexity, degeneracy, evolvability, redundancy, and robustness are important in the study of high-dimensional, complex biological systems. As his 2012 paper in Journal of Theoretical Biology illustrates, Yao’s work focuses on the quantification of certain systematic measures for networks that are modeled by ordinary differential equations. Based on estimates of the weak noise limit of stochastic differential equations, a rigorous mathematical framework for complexity, degeneracy, and robustness is established in two of his papers that will appear in Communications on Pure and Applied Mathematics. In future research Yao will study the quantification of certain systematic measures for spiking neuron models. The fundamental goal of this challenging study is to establish a connection between systematic measures and network structures. Such a connection will benefit both systems biology and mathematics itself.

Yao’s other research field related to mathematical biology is the numerical and analytical study of non-equilibrium neuron field models. In particular, he and his collaborators work on the human primary visual cortex (V1). The complicated function of V1 is achieved by both the interplay of excitatory and inhibitory neurons and the external visual signal that drives the system out of its equilibrium. Therefore, a V1 model resembles a statistical mechanics model in many aspects. Yao and his collaborators are working on the development of fast stochastic simulation methods — as shown in his 2015 paper in The Journal of Chemical Physics — and the large-scale simulation of human primary visual cortex. The immediate goal is to reproduce, by numerical simulations, experimentally observed phenomena such as Gamma waves, surround suppression, and orientation-tuning curves. Besides his study of numerical methods, Yao is working on the analytical study of the stochastic stability of a class of V1 models.
NEW FACES IN THE DEPARTMENT

Patrick Flaherty joined the department as an assistant professor in September 2015. He works on statistical methodology for large-scale genomic data analysis. In the past ten years, the cost to sequence the DNA of the human genome has gone from $10 million to just over $1,000. As a result, we are now able to collect very large genomic data sets for a reasonable cost. These data sets, when combined with rigorous analysis tools, enable us to better understand and treat complex genetic diseases such as cancer. Patrick’s group develops statistical methods to interpret and integrate these large genomic data sets.

Patrick completed his B.S. in electrical engineering at Rochester Institute of Technology. He earned his Ph.D. in electrical engineering and computer science at University of California, Berkeley with Professor Michael Jordan (computer science & statistics) and Professor Adam Arkin (bioengineering). He completed a postdoctoral fellowship in the Departments of Biochemistry and Genetics at Stanford University with Professor Ronald Davis. He was an assistant professor at Worcester Polytechnic Institute for three years before joining the department at UMass Amherst.

Yao Li joined the department as an assistant professor in September 2015. His primary research interest is in applied dynamical systems and applied probability, with a focus on problems arising in mathematical physics and mathematical biology. His main interests in mathematical physics are the ergodic and limit properties of non-equilibrium steady states of statistical mechanics models, where “non-equilibrium” means that a system is irreversible and violates the symmetry condition known as detailed balance. In mathematical biology, his work focuses on the quantification of systematic measures of complex biological networks and the study of non-equilibrium neuron field models.

Yao completed his B.S. at Shanghai Jiao Tong University in 2007. He earned his Ph.D. in the School of Mathematics at Georgia Institute of Technology under the supervision of Professor Yingfei Yi. Before joining the University of Massachusetts Amherst, he was a Courant Instructor at the Courant Institute of Mathematical Sciences at New York University.

RECENT FACULTY PROMOTIONS

At the end of last summer, the UMass Board of Trustees approved the recommendation of the Provost and Chancellor to promote Senior Lecturer Jeff Beaulieu to the rank of Senior Lecturer II, and to promote Lecturer Jinguo Lian to the rank of Senior Lecturer, effective 9/1/15.

Jeff Beaulieu joined our department as a Lecturer in 1999 and was promoted to Senior Lecturer in 2008. Beaulieu makes valuable contributions to teaching courses in business calculus, precalculus, and math for elementary school teachers, and he also mentors teaching assistants. He currently serves on the General Education Council and the Faculty Senate.

Jinguo Lian joined our department as a Lecturer in 2009. For quite a few years, he has rendered very valuable service to the department as the course chair for calculus I and II and for differential equations. In recent years, he has become involved in teaching courses for students in the actuarial concentration and served as Co-Director of the Actuarial Program this past year.

Provost Katherine Newman and Chancellor Kumble Subbaswamy have enthusiastically recommended to the UMass Board of Trustees to promote Assistant Professor Krista Gile to the rank of Associate Professor with tenure. After earning a B.S. in Electrical Engineering at RPI in 1998 and an M.S. in science and technology studies at Virginia Tech in 2000, Gile obtained a Ph.D. in statistics in 2008 at the University of Washington under the direction of Mark Handcock. She held a postdoctoral position at Oxford from 2008 to 2010. She then joined our department in the fall of 2010 as an Assistant Professor. An expert in the statistical analysis of social networks, her work in respondent-driven sampling has earned her international renown. Over the last few years, Gile has expanded her research far beyond the realm of RDS methodology; her activities include collaborations with multiple researchers across campus in sociology and computer science.

Congratulations to Jeff, Jinguo, and Krista for these well-deserved promotions!

FACULTY PROFILE: ANDREA NAHMOHD

Professor Andrea Nahmod’s interest in mathematics began in early childhood. Her fondest memories include sitting in her mother’s kitchen as an elementary school child doing her math homework. “I found it very peaceful,” says Nahmod.

Growing up in Argentina, Nahmod found the logic of mathematics a safe harbor in the political upheaval of the 1976-1983 military coup and dictatorship that threw the country into turmoil during her high-school and first university years. “Things were so oppressive during that period in our history,” says Nahmod. “Math was a place where things were logical. It made sense to me at a time when what was happening around me made no sense.”

Having come from a family of medical researchers and physicians, Nahmod decided early on to study medicine. But her initial focus on how mathematics applied to genetics, as well as a key conversation with a former neighbor who worked for NASA and the Smithsonian Astrophysical Observatory, led her to reconsider her choice. “He showed me how math was the basis of many other disciplines,” says Nahmod, “and encouraged me to do math as a career. Somehow he opened my head for math, and I was convinced.”

It turned out to be good advice. An expert in the two separate but interrelated classical areas of mathematics called harmonic analysis and partial differential equations, Nahmod has risen to national prominence in her field. Since receiving her PhD from Yale University in 1991, she has received many prestigious honors: a Radcliffe-Sargent Fulbright Fellowship in 2009 took Nahmod to the Radcliffe Institute at Harvard University for a year to expand her research program; she was selected for a highly competitive Simons Fellowship in 2013 allowing her a full-year sabbatical at MIT; and she’s been twice selected a Member of the Institute for Advanced Study at Princeton. In 2014 Nahmod was one of a select few mathematical scientists from around the world to be named a Fellow of the American Mathematical Society in recognition of her contributions to nonlinear Fourier analysis, harmonic analysis, and partial differential equations, as well as service to the mathematical community.

Most recently, Nahmod received a Simons Professorship at the Mathematical Science Research Institute in Berkeley, California, where she also co-directed an international effort in which more than 200 researchers worked in concert on a set of problems in partial differential equations. Additionally, the National Science Foundation recently awarded Nahmod and collaborators a $1.4 million Focus Grant to fund their research for the next three years. “This continues her sterling record of NSF funding in every year that she has been at UMass Amherst,” says Farshid Hajir, professor and head of UMass Amherst’s Department of Mathematics and Statistics.

When asked about her research, Nahmod says, “I’m an analyst. I study how to decompose objects into forms that we can understand and that give us information about their most relevant features, their structure and patterns.” Using techniques called harmonic and nonlinear Fourier analysis, Nahmod and her colleagues apply these decomposition techniques to problems in the material world in order to find solutions and to understand their behavior.

“We can break down images and signals such as speech, radar, or wave propagation in optics into modulated waveforms, the signal’s basic building blocks which capture their main features and are easy to compute,” says Nahmod. “At the same time this gives us a way of putting them back together using only some parts without losing the signal’s basic qualities. It’s an important process that has revolutionized digital technology.” This ability to compress signals into wave packets has been helpful when reconstructing digital fingerprints or developing face-recognition software, two technologies important to public safety and national security.
Nahmod’s area of expertise is in nonlinear partial differential equations (PDEs) modeling different wave-propagation phenomena in nature. Nonlinear wave models arise in quantum mechanics, fiber optics, ferromagnetism, water waves, Bose Einstein condensates, and many other phenomena. One of the most ubiquitous of these PDEs is the nonlinear Schrödinger equation (NLS).

“Because waves in nature interact in a nonlinear fashion as they propagate and have different properties such as amplitude, length, oscillation, speed, and position over time, it’s important to understand how they may behave under certain conditions or when introduced to certain media,” says Nahmod. “Understanding the most efficient way to send a signal through a fiber optic cable or being able to anticipate the properties of a gas when the temperature approaches absolute zero are two very different phenomena in nature but are both aspects of solutions to the same equation,” she adds.

“Being able to understand and describe the dynamics and behavior of solutions to NLS is fundamental to accurately predict wave phenomena,” says Nahmod. “That’s an important tool to have in your toolkit when studying the natural world.”

What’s next for her research? Nahmod has always been interested in bringing new tools to bear on current mathematical methods in order to open up new research directions or define new paradigms. “I like being able to approach problems in new ways,” she says. “To move the problem forward in a different fashion. That is a strength of mine and of the faculty at UMass Amherst.” To this end, her current research investigates the role of data randomization in nonlinear wave phenomena and how probability can be applied to shed light on behavior and dynamics of generic solutions.

“Roughly,” Nahmod explains, “the idea is that you don’t need to look at the dynamics of equations for every single initial profile of a class to predict an outcome. Probability introduces the notion that you can look at it generically: pick one at random that has certain prescribed properties, and understand the long-term dynamics of it by approaching it from a non-deterministic viewpoint.”

Two attractions drew Nahmod to UMass Amherst: it gave her the opportunity to teach at a public institution, and it introduced the notion that you can look at it generically: pick one initial profile of a class to predict an outcome. Probability need to look at the dynamics of equations for every single solution.

As a product of the public school system in Argentina, Nahmod appreciates the role public universities such as UMass Amherst play in citizens’ lives. “There is something that I like that is a little bit of a challenge at a public school,” she says. “The student body is larger and more diverse and includes an enormous number of bright kids. To find them and tap their potential, to reach out to them is rewarding. I was helped in that way when I was their age. It’s time to pay it forward.”

**Editors’ Note:** This article is a lightly edited version of the article written by Karen J. Hayes ’85 on the occasion of the Spotlight Scholar award to Andrea Nahmod. Her article can be accessed at http://www.umass.edu/research/exhibit/spotlight/mathematical-mind. Karen is the Director of Research Communications and Marketing at UMass Amherst.

Haitian Yue were named Program Associates. All three were in residence for the duration of the program. Assistant Professor Yao Li was a member in residence for one month, and Nahmod’s Ph.D. students Michael Borakto and Jiabui Yu participated in the thematic workshops. In addition, incoming Assistant Professor Sohrab Shahshahani was awarded an NSF postdoc for the duration of the program.

The program included three programmatic workshops. Professor Nahmod was the lead organizer of the two-week research workshop titled “New Challenges in PDE: Deterministic Dynamics and Randomness in High and Infinite Dimensional Systems,” which took place October 19–30, 2015, and an organizer of the workshop titled “Connections for Women: Dispersive and Stochastic PDE” which took place August 19–21, 2015. In addition Professor Nahmod delivered three lectures at the “Introductory Workshop: Randomness and Long Time Dynamics in Nonlinear Evolution Differential Equations,” which took place August 24–28, 2015. These three lectures were streamed live, and video recordings can be found at https://www.msri.org/people/2846.

The program also ran weekly activities that included research seminars, research mini courses, postdoc seminars, graduate student seminars, and focused topical meetings by various research teams.

During the past twenty years the theory of nonlinear dispersive partial differential equations and systems (PDE) and the theory of stochastic partial differential equations (SPDE) have each developed into a mature mathematical theory. On the PDE side, the development of analytical tools in nonlinear Fourier and harmonic analysis to address nonlinear estimates, related deep functional analytic methods, and profile decompositions have fundamentally contributed to the study of the long-time dynamics as well as singularity formation for dispersive equations and systems. This body of work has focused primarily on deterministic aspects of wave phenomena. More recently, however, several publications have appeared that aim at understanding the non-deterministic point of view as well. On the SPDE side, similar questions about existence, uniqueness, and qualitative long-time behavior have also been addressed for a large number of models by bringing together tools from statistical mechanics, dynamical systems, and probability theory.

In recent years, the focus in both PDE and SPDE has thus shifted to trying to gain a more quantitative understanding of the non-deterministic long-time dynamical behavior in various regimes. While there has been spectacular progress in both of these fields, the advances have taken place in a parallel fashion without a substantial exchange of ideas between the two mathematical communities. However, many fundamental questions still remained open in both fields. The organizers of this research program believed that these questions could be tackled by exploring the numerous connections between them.

Professors Nahmod and Rey-Bellet and the other co-organizers envisioned this research program to be a venue where experts on dispersive and wave equations, on stochastic equations, and on the intersections of these two major fields of research would converge in a single institution to discuss and to tackle some of the most challenging questions that remain unanswered in these fields and to develop new integrative methods.

By all accounts the program was a tremendous success. Conference details are available at https://www.msri.org/programs/287.

**NSF Grant for Cybersecurity at UMass Amherst**

In January 2015 a team of cybersecurity researchers at the University of Massachusetts Amherst received a $4.2 million grant from the National Science Foundation to bring a CyberCorps Scholarship for Service program to the campus, the first public university in New England to receive such an award. The team, which brings together researchers from the College of Information and Computer Science, the Department of Mathematics and Statistics, the Department of Electrical and Computer Engineering, and the Isenberg School of Management, includes Professor Eric Sommers, who is a co-PI on the grant, and Professor Krista J. Gile.

NSF’s CyberCorps program, in partnership with the Department of Homeland Security, supports the educational and professional development of domestic students who will help the nation address threats to national security including critical infrastructure such as utilities, water treatment, military defense systems, and refineries.

Upon graduating and completing the training, students will join, at full pay and benefits, government agencies working in cybersecurity, such as the National Security Agency, the Department of the Navy, the Department of Treasury, the FBI, and other agencies at the federal, state, or local level. Any government service in cybersecurity fulfills the service requirement, ranging from protecting the nation’s infrastructure from state-based hackers to joining a state university as a researcher or educator in cybersecurity.
The UMass program, which will support a total of 28 students over the next five years, will admit its first students in the fall 2016 semester. Students must be U.S. citizens or permanent residents and can receive up to two years of support from the CyberCorps program. For each year they accept aid, they will serve for a year in a federal, state, or local government position related to cybersecurity.

The Department of Mathematics and Statistics expects to receive one scholarship per year to build up a cohort of students with expertise in both security (through coursework in computer science) and expertise in mathematics and statistics (through coursework in number theory, modern algebra, statistics, and other research fields represented by the department). Many members of the department helped with the NSF site visit in November 2015. In addition to Gile and Sommers, undergraduates Gabriel Andrade and Shelby Cox, graduate students Dongah Kim and Dan Nichols, and Professors Farshid Hajir and Siman Wong participated in the site visit. Moreover, Andrade, Kim, and Nichols presented posters on their research at a poster session held during the site visit.

Additional information on this important grant and the CyberCorps Scholarship for Service program is available at http://www.uml.edu/colleges/sciences/cybercorps/.

COMPUTATIONAL SOCIAL SCIENCE

Data are becoming an increasingly important part of understanding the world we live in. Social science is no exception. What do you think of when you think of social science research? An anthropologist observing a remote village? An election pollster? Perhaps a psychologist in a lab presenting a child with the following choice: one marshmallow now, or wait to eat it and receive a second one if we remove all fractions which contain a digit 3 in base 10? Although these may seem simple, they reveal an important underlying principle, foundational in modeling social and political processes and the distribution of wealth. These types of problems are at the heart of a large area of research in mathematics known as discrete mathematics, which is a branch of combinatorics, a part of mathematics and statistics that studies the properties of finite or countable objects. These objects can be simple elements like the 0's and 1's in computers, or complex objects like social networks or the arrangement of molecules. In this section, we will briefly describe some of the fundamental concepts and applications of discrete mathematics, showing how these seemingly innocuous problems can be used to understand the real world and solve practical problems.

The Computational Social Science Institute at UMass Amherst (www.cssi.umass.edu) brings together researchers taking advantage of these exciting opportunities. Computational social science research often emerges from connections between social scientists with important questions to understand and technical scientists interested in developing novel methods. The Institute facilitates such connections, especially among its 68 faculty affiliates and many involved students from 23 departments in 9 colleges at UMass. It hosts a weekly seminar series with expert speakers from UMass, across the country, and from around the world, as well as workshops, mixers, and events to connect researchers throughout the university.

Statistics, and also applied math, are of great use in computational social science, and the Department of Mathematics and Statistics is one of the four founding departments of the Computational Social Science Institute. Faculty members Krista Gile, Michael Lavine, and John Staudenmayer are also faculty affiliates. Professor Gile's research includes developing network-based survey methods to survey hard-to-reach, high-risk populations like people who inject drugs. Professor Lavine's research includes analyzing experimental data collected during brain surgery to understand and treat the patterns underlying epilepsy. Professor Staudenmayer's research includes increasing our understanding of accelerometer data: how many hours was I sedentary today?

These are only a few of the many fascinating and important computational social science topics studied by faculty and students in the department and at UMass. Keep watch for many exciting developments in this area, and check out the Computational Social Science Seminar Series, free, open to all, and offering free lunch!
work in base 2 instead of base 10 and remove one digit, say zero. The integer numbers we obtain using only the digit 1 in base 2 are 1, 11, 111, 1111, 11111, and so on.

\[ 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 + 1 = 2^{19} - 1 \]

From here is seems pretty clear what the answer should be and how to verify it.

**Problem 4.** What is the chance that exactly 2 people in a family of 3 were born on the same day of the week (assuming it’s equally likely to be born on any day of the week, and there are no multiple births)?

The probability that one family member is born on a given day, say Monday, is 1/7. Thus, the probability not to be born on Monday is 6/7. Therefore the probability for family members 1 and 2 to be born on Monday and family member 3 not to be born in Monday is (1/7)² × 6/7. Since the question is not about being born on Monday, but on an arbitrary day of the week, we obtain that the probability for family members 1 and 2 to be born on the same day of the week and family member 3 not to be born on that day is 1/7 × 6/7. Again, the question is not about two particular family members in a family of three being born on the same day. Therefore we need to multiply by the number of two element subsets in a set of three elements, that is \( {3 \choose 2} = 3 \), and obtain 18/49 as the probability that exactly two family members in a family of three are born on the same day of the week.

Such arguments leave a lot to be desired since the general context in which they take place is not apparent. For example, we could have asked: “What is the probability that out of 17 people 3 have their birthdays on Monday, another 5 have their birthdays on Friday, and yet another 2 have their birthdays on the same day but neither Monday nor Friday?” or any number of similar questions. A conceptually more satisfying approach arises when one considers such questions as a counting problem of configurations, that is, of maps between finite sets. Let us write \( [n] \) for the set \{1, ..., n\} with \( n \) elements. Then a configuration is a map \( x: [k] \to [n] \). In our challenge problem \( k = 3, n = 7, x(1) = 1 \) is interpreted as “family member 1 has birth day on day 1 of the week.” In order to calculate the probability that exactly 2 people in a family of 3 were born on the same day of the week, we need to count the number of configurations \( x \) such that for some \( j \in [7] \) the preimage \( x^{-1}(j) \) has exactly two elements. Then we divide this number by the number of all configurations \( n^k \), which is 7 in our problem. We next apply what is probably the most important idea in mathematics (and the sciences in general), namely symmetry. Using the permutation group \( S_n \) on the range \( [n] \) and \( S_k \) on the domain of \( x \), we can always arrange for \( j = 1 \) and \( x(1) = 1 \) (2). Thus, we are led to count the number of \( g \) for \( x(1) = 1 \) and \( x(2) = 1 \) and \( x(3) = 2 \), \( g \in S_3 \), and \( h \in S_7 \), that is, the number of elements \( n \) in an orbit of \( x \) under the product group \( S_n \times S_k \times S_7 \). At first sight this appears to be counting the number of elements in \( S_n \times S_k \times S_7 \) which is \( 7! \times 3! \). But we notice that there are elements \( g, h \) for which \( x(1) = x(2) = x(3) \), which stabilize \( x \). For instance, \( g \in S_3 \) permuting only the elements 1, 7, 7, and the permutation \( h \in S_7 \) exchanging 1 and 2 and fixing 3 will stabilize \( x \). These are exactly the stabilizer subgroup \( S_n \times S_k \times S_7 \). The orbit through \( x \) is therefore given by the coset space \( S_n \times S_k \times S_7 \). Let \( S_7 \) have 71 + 35 + 21 + 15 + 10 + 6 + 3 + 1 = 120 elements. Therefore there probability that exactly 2 people in a family of 3 were born on the same day calculates to 6 × 7 × 3! = 18.49. With these tools at hand the reader can now start on the problem “What is the probability that out of 17 people 3 have...” mentioned above and, for that matter, on any of those counting problems.

We have seen a glimpse that the notion of a symmetry group acting on a set is crucial for a more conceptual approach to the problem. This idea permeates all of modern mathematics, with Galois groups, Lie groups, and Klein’s Erlangen Program (a geometry defined by its symmetry group) being the most famous historical examples.

**Problem 5.** Five mathematicians – Alex, Franz, Jenia, Paul and Bob – sit around a table, each with a huge plate of cheese. Instead of eating it, every minute each of them simultaneously passes half of the cheese in front of him to his neighbor on the left and the other half to his neighbor on the right. Is it true that the amount of cheese on Franz’s plate will converge to some limit as time goes to infinity? Is it true that the amount of cheese on Paul’s plate will converge to some limit as time goes to infinity? Is it true that the amount of cheese on Franz’s plate converges to some limit as time goes to infinity?

This is a cheeky version of discrete heat flow with spatial periodic boundary conditions and heat conductance 1. Heat flow is one of the most fundamental models in nature. Its stationary solutions are harmonic functions, already used by Riemann and later Hodge, to prove deep results about the structure of complex manifolds; heat flow with imaginary conductance is the Schrödinger equation; nonlinear heat flow such as the Ricci flow involves geometry and was used by Perelman to prove the Poincaré conjecture. Now to our discrete cheese flow: let \( f(t) = f_1(t), f_2(t), \ldots, f_n(t) \) denote the amount of cheese of person \( k \) at time \( t \) measured in minutes. Then the vector \( f(t) = \{f_1(t), f_2(t), \ldots, f_n(t)\} \) evolves via

\[
f(t + 1) = \mathbf{A}f(t)
\]

for a 5 × 5 symmetric matrix \( \mathbf{A} \) with non-zero entries \( A_{ij} = 1/2 \) if \( i = j + 1 \) or \( i = j - 1 \) (where we interpret the indices modulo 5). Thus, \( f(0) = \{0, 0, 0, 0, 0\} \) encodes the initial amounts of cheese on each of our mathematician’s plates. Let us assume the initial total amount of cheese is 1 kilogram. Since each row sum of \( \mathbf{A} \) is 1, we see that the total amount of cheese \( f(1) = f_1 + f_2 + \ldots + f_5 \) is independent of time (as we expect as long as our mathematicians do not crumble cheese onto the table...), that is, \( |f(t)| = |f(0)| = 5 \). Since \( A \) is a symmetric matrix it can be diagonalized and all of its eigenvalues \( \lambda \) are real. In fact, since all of the row sums of \( \mathbf{A} \) are 1, each eigenvalue has magnitude at most 1. The vector \( v = (1, 1, 1, 1, 1) \) is an eigenvector with eigenvalue \( \lambda = 1 \), and it is easy to check that \( v \) is the only (up to scale) eigenvector corresponding to the eigenvalue 1. Thus, all the other four eigenvalues have magnitudes strictly smaller than 1. Now write the initial cheese distribution \( (f(0)) = (a_1, a_2, a_3, a_4, a_5) \) in an eigenvector for \( \lambda \).

\[
\lim_{t \to \infty} f(t) = \lim_{t \to \infty} \mathbf{A}^t f(0) = \sum_{k=0}^{\infty} (\mathbf{A}^t)^k f(0)
\]

since \( A_{ij} = 1 \) and \( |A| < 1 \) for \( k \geq 2 \). But the total amount of cheese is preserved and so \( |f(t)| = 1 \) or \( |f(t)| = 1/5 \). So yes, Franz and each of the other mathematicians around the table will eventually have 1/5, that is 200 grams, of the initial total amount of cheese on their plates. As it turns out, Franz doesn’t really care for cheese that much and did not participate in that cheese flow experiment. Would the four remaining mathematicians carrying out the same experiment eventually be left with each 1/4, that is 250 grams, of the initial amount?

**Problem 6.** Cut a round disk into the greatest possible number of pieces using 6 straight lines.

The first cut makes at most 2 pieces, the second cut adds at most 2 more pieces, and thus the \( n \)-th cut adds at most \( n \) pieces resulting in at most

\[
2 + \sum_{k=1}^{n} k = 1 + \sum_{k=1}^{n+1} k = \frac{n(n + 1)}{2}
\]

pieces when cutting \( n \) times. Christopher Thomas’ drawing shows that one can actually get that many pieces.
OUTSTANDING STUDENTS HONORED AT 2016 AWARDS DINNER

On April 5, 2016, in the Amherst Room of the Campus Center, more than 100 people gathered to celebrate the achievements of our undergraduate majors and graduate students. The occasion was the annual Awards Dinner of the Department of Mathematics and Statistics, whose scope was greatly expanded from previous years to honor 54 students in 18 different categories. Besides faculty, students, family, and friends, those in attendance included Dean Steven Goodwin, alumni James Francis ’86, Roy Perdue ’73, and Robert Reitano ’72, and Professor Emeritus Eleanor Killam.

As our guests savored desserts of apple crisp and ice cream, the awards portion of the event opened with greetings from our Department Head, Professor Farshid Hajir. Both Farshid and Richard S. Ellis, the organizer of this year’s Awards Dinner and emcee, thanked our donors for their support of the department, as well as for making possible a number of the awards and activities recognized at the dinner. Richard also thanked Farshid, Christine Ingraham (Assistant to the Department Head), and Professor Paul Hacking (last year’s Awards Dinner organizer and emcee) for their help with various aspects of this year’s dinner.

Three math puzzles were included in the program; two word problems involving no mathematical notation, and a literary reference problem to the work of Franz Kafka for those less interested in math puzzles. The full program can be viewed at http://www.math.umass.edu/sites/www.math.umass.edu/files/file-attachments/awards-dinner-program.pdf.

After the awards ceremony, Dean Steven Goodwin made the closing remarks, praising the department both for the quality of its faculty, undergraduate majors, and graduate students and for the quality of this year’s Awards Dinner, which unfolded like a perfectly choreographed dance.

Undergraduate Awards

The range of activities and the number of awards presented to our undergraduates are impressive. Faculty introduced the students and gave background on their activities and awards.

Four members of Commonwealth College – Alon Gelber, A. Kader Geraldo, Daniel Normand, and Zachary Plummer – were recognized by Professor Farshid Hajir for Honors Research and Theses under the direction of a faculty member.

The Putnam Competition is the oldest and most prestigious mathematical competition for undergraduate students in the U.S. and Canada. Thomas Bogue, Stephen Obinna, and Aubrey Wiederin were recognized for being members of the Putnam Competition Team. Professor Ivan Mirkovic, who ran the Putnam preparation seminar, introduced the students.

Thanks to the generous support of Joan Barksdale (’66) and faculty research grants, the department has been able to significantly expand its summer REU (Research Experience for Undergraduates) Program. Three faculty members introduced the participants, described last summer’s REU projects, and explained the basics of our REU program.

Professor Farshid Hajir introduced the students who participated in the REU in Pure & Financial Math, recognizing Aaron Dunbrack, Alex Lassalle, Makai McClintock, Kai Nakamura, Daniel Normand, Stephen Obinna, Cristian Rodriguez, Ryan Ross, and Aerin Thomson.

The REU in Applied Math was presented by Professor Panos KEVREKIDIS, who recognized Sagiv Ben-Ari, A. Kader Geraldo, Soohyun Kim, and John Lee. Kenyon Lewin Berlin, who participated in the REU in Statistics, was honored by Professor Anna Liu.

Of the nine Undergraduate Awards, the first two have been awarded for a number of years while the last seven are new and were initiated during this Awards Dinner. Each faculty member who introduced the students winning each award summarized the students’ backgrounds and explained why each one was chosen as a recipient. Each student received a personalized certificate marking his or her achievement.

The M. K. Bennett Geometry Award is presented to the student who exhibits the best performance in Math 461. This award honors the memory of Professor Mary Katherine Bennett, who earned the first Ph.D. in our department in 1966. After teaching at Dartmouth College, she returned to UMass Amherst for the rest of her career where she encouraged interest in geometry and high school teaching among undergraduates. The year-long course that she developed now covers Euclidean, spherical, and hyperbolic geometry, and it is taken by all our math majors in the teaching concentration.

Professor Paul Hacking introduced Stephen Obinna, the winner of the M. K. Bennett Geometry Award, and two more recipients.

Awards Dinner photo credit: Vivian Bresnitz
runners-up, Daniel Normand and Cristian Rodriguez, each of whom received honorable mention. Three of these students were selected by Professor R. Inanc Baykur, who taught Math 461 during the fall semester.

The Jacob-Cohen-Killam Mathematics Competition is named in honor of the memories of Professors Henry Jacob and Haskell Cohen and in honor of the continuing contributions of Professor Emeritus Eleanor Killam. The three faculty members sparked interest in mathematics among undergraduates through annual mathematics contests.

The competition is open to first-year and second-year students. Each year a few dozen contestants attempt to solve ten challenge problems proposed by faculty members. Former contestants often develop deep ties with the department; some choose mathematics as a major while others participate in summer REUs or join the Putnam team. This year the competition was again generously sponsored by John Baillieul ’86, James Francis ’86, and Roy Perdue ’73.

This year’s first prize was shared by Thomas Bogue, Ka Min Nakamura, and Aubrey Wiederin. The second prize went to James Hagborg. The prizes were presented by Professor Franz Pedit.

The next four awards are new. The Don Catlin Award for Outstanding Achievement in Applied & Computational Mathematics recognized Alon Gelber, A. Kader Geraldo, and Brooke Herzog; it was presented by Professor Panos Kevrekidis.

The Bob and Lynne Pollack Award for Outstanding Academic Achievement in Actuarial Science, presented by Professor Eric Sommers, recognized the winner, Chris Jacob, and the runners-up, Augustus Lord and Brian Penta, each of whom received honorable mention. Eric is the Chief Undergraduate Advisor and has been instrumental in building up the actuarial program in the department.

Professor Farshid Hajir presented the Steve and Geni Moncharmont Student Leadership Award to Shelby Cox, Athena Higgins, and Augustus Lord, and the Award for Outstanding Academic Achievement in the Teaching Concentration to Erica Doyle and Amanda Schleicher. The last three awards, which are also new, recognize Outstanding Academic Achievement for Sophomores, Juniors, and Graduating Seniors. Three sophomores, Shelby Cox, Michael Shilsberg, and Jeffrey Spahli; three juniors, Aaron Dunbrack, Radha Dutta, and Cristian Rodriguez, and four graduating seniors, Gabriel Andrade, Scott Destromp, Azerin Thomson, and Artur Wysoczanski, were presented these awards by Professor Eric Sommers.

Graduate Student Awards

Professor Tom Weston, the Graduate Program Director, presented four Graduate Student Awards to five students. The first two awards have been presented for a number of years while the last two are new this year. While presenting each award, Tom summarized the student’s background and explained why each student was chosen as a recipient of the award. Each student received a personalized certificate marking his or her achievement. Details about each award are given in the second half of the article ”Graduate Student News” in the section titled ”Thesis, Teaching, and Leadership Awards.” See page 26-27.

The winners of the Distinguished Thesis Award were Nikolaus Buskin and Haitao Xu. Jiuhui Yu received the Distinguished Teaching Award. The winner of the Five College Teaching Fellowship was Andrew Havens. Konstantinos Gourgoulianis was the recipient of the Graduate Student Leadership Award.

RESULTS OF THE 2015 ANNUAL PUTNAM COMPETITION

The Putnam Mathematical Competition is the most prestigious mathematical competition for undergraduate students in the United States and Canada. In 2015 there were 4,275 participants from 554 colleges and universities. The winning teams were from MIT, Carnegie Mellon, Princeton, Stanford, and Harvard. Each school selects a three-person team; the UMass Amherst team members were Thomas Bogue, Stephen Obinna, and Aubrey Wiederin.

The examination takes six hours separated by a lunch break. The participants are given twelve problems, which rely on the standard undergraduate curriculum. However, they require sophisticated investigations and delicate reasoning. The competition in 2015 was unusually challenging. There was a problem that nobody solved and another problem on which only eight participants received more than 20%. In addition, two-thirds of participants did not solve any problems.

Our department offers a one-credit course, Math 491A, which is a preparation for the Putnam competition. During the fall semester of 2015 it was taught by Professor Ivan Mirkovic. The course is very dynamic as each week covers a different area of mathematics including combinatorics, game theory, geometry, graph theory, induction, linear algebra, number theory, polynomials, probability, and real analysis. Students generally enjoy this rhythm, which emphasizes use of their total mathematical experience to think problems through. This course also functions as a community-building mechanism that connects mathematics enthusiasts at UMass. Various aspects of the course have been developed mostly by Professor Jenia Tevelev, who volunteered to teach the course five times. One can find out more about this course at http://people.math.umass.edu/~tevelev/putnam/putnam2014.html.

In the 2015 competition seventeen UMass students took the examination. The UMass participants with distinguished performance were Ji-Hun Hwang, Ka Nakamura, Russell Phelan, Austin Sieberlich, and the three UMass team members mentioned in the first paragraph. An Amherst College student, Bowen Yang, also took the preparatory UMass course and placed among the first 350 participants.

Problems, solutions, and winners of recent competitions can be found at http://kskedlaya.org/putnam-archive/.

ANDREW HAVENS RECOUNTS EIGHT GREAT YEARS OF THE UNDERGRADUATE MATH CLUB

Eight years ago, as an undergraduate at UMass Amherst, I discovered the Math Club. For a time, it was a wonderful secret: talks by graduate students and occasionally faculty on a panoply of problems in mathematics, with an abundance of pizza for the few of us who knew to show up each week. Once in a while, a game such as Set was brought out during a slow week around exam times or near the end of the semester. This past fall, I joined fellow graduate student Dan Nichols in coordinating the current iteration of Math Club talks and activities. As we reach out to colleagues in search of talks and plan for ways to engage the club with the sort of mathematical play that too often is missing during the semester, I’m reminded of those undergraduate days of which I’m so fond, when, driven by curiosity and a thirst for novel math, I chose to linger a little longer in the Lederle Graduate Research Tower.

During most weeks, the Math Club gives undergraduates a taste of seminar learning and a chance to ask questions about an area of active research. Since September, the club has hosted talks about such topics as curvature and geometry of surfaces, ecosystem dynamics, graph theory, group theory, low dimensional topology, public key cryptography and elliptic curves, and statistical inference with social network data. With the help of undergraduate speakers, the Math Club has also learned about the Research Experiences for Undergraduates program. In addition to talks, we coordinate activities some weeks. The club this year has been introduced to some of Conway’s favorite games such as Sprouts, GOPS, and SWAPS, to logic puzzles such as Cave and Slitherlink, and to classic videos such as the Geometry Center’s ”Outside In,” the topic of which is sphere eversion. The biggest difference between the Math Club of my memories eight years ago and the Math Club of today is attendance. The club has grown considerably, with regular attendees filling the seats of our regular room during most weeks. It is my hope that year after year, long after Dan and I have defended our theses, young students brimming with enthusiasm and curiosity about the reaches of mathematics will continue to come each week to the Math Club.
Six students received Ph.D. degrees between September 2015 and May 2016: Nikolay Buskin, Mei Diau, Stephen Oloo, Evan Ray, Peng Wang, and Tobias Wilson. In addition, eighteen students received M.S. degrees: Victoria Day, Chaqon Jia, Vy Nguyen, Jiahui Yu, Xuezeng Yu, and Haitian Yue in mathematics; Simon Burhoe, Emma Dowling, Peng Du, Rachel Gordon, Thamanya Saksuriyongse, and Ankita Shankhardar in applied mathematics, and Betsy Camano, Doruk Cengiz, Osman Iciz, Dongah Kim, Yan Ling, and Matthew Schmidt in statistics. Congratulations to all of these students!

In May Isabelle Beaudry gave an invited talk on “Advances in Key Population Link-Tracking Network Surveys” at the 3rd Global Consultation Meeting on HIV Surveillance (organized by WHO and UNAIDS) in Bangkok. She also submitted a paper titled “Inference for Respondent-Driven Sampling with Misclassification” and presented this work at the International Statistical Institute Satellite Meeting on Small Area Estimation in Santiago, Chile; at the New England Statistical Symposium at UConn Storrs; and at the Social Science Research Beyond the Academy Conference at UMass Amherst, where she received an award for most creative methodology. The trip to Chile was supported by an NSF travel grant. Finally, Isabelle presented work on “Correcting for Differential Recruitment in Respondent-Driven Sampling” at the Joint Statistical Meetings in Seattle. She will begin a tenure-track position at Pontificia Universidad Catolica de Chile starting in March 2017.

Konstantinos Gourgoulias, jointly with Professors Markus Katsoulakis and Luc Rey-Bellet, submitted the paper titled “Information Metrics for Long-Time Errors in Splitting Schemes for Stochastic Dynamics and Parallel KMC” to the SIAM Journal on Scientific Computing. He also attended the Predictive Multiscale Materials Modelling Workshop at Cambridge University, UK and presented a poster there on “Information Metrics for Splitting Schemes for Parallel Kinetic Monte Carlo.”

Konstantinos Gourgoulias gave a talk titled “Asymptotic Behavior of the Invariant Density of the Random Logistic Model” at Applied Math Days 2016, a seminar for postdoctoral fellows and graduate students held at Rensselaer Polytechnic Institute in Troy, NY.


This year five awards were given to graduate students in our department. The winners of the Distinguished Thesis Award were Nikolay Buskin and Haitao Xu. Jiahui Yu was the recipient of the Distinguished Teaching Award. The winner of the Five College Teaching Fellowship was Andrew Havens. Konstantinos Gourgoulias was the recipient of the Graduate Student Leadership Award.

Nikolay Buskin works in the field of algebraic geometry under the supervision of Professor Eyal Markman and is one of two recipients of our Distinguished Thesis Award. His thesis proves important new cases of the Hodge conjecture for products of K3 surfaces. The Hodge conjecture, which is one of the seven Clay millennium problems, predicts which cohomology classes of algebraic varieties arise from geometry. Nikolay’s work has already been very influential and has been studied extensively by algebraic geometers throughout the world.

Haitao Xu works in applied mathematics under the supervision of Professor Panos Kevrekidis and is one of the two recipients of our Distinguished Thesis Award. His thesis studies two primary topics related to systems of nonlinear ordinary differential equations: elastic chains of beads interacting via nonlinear pairwise interactions and parity-time symmetric nonlinear systems. In each case he establishes fundamental existence results and computationally analyzes the behavior. He provides a definitive analysis of the particular waves he studied for the chains (called nano-teron waves) at essentially the same time they were observed experimentally.

Jiahui Yu works in analysis under the supervision of Professor Andrea Nahmod and is the recipient of our Distinguished Teaching Award. She has taught calculus as well as differential equations numerous times. Her students praise her clear, efficient, precise, thorough, and informative teaching style as well as her clear explanations and efforts to make examples relevant.

Andrew Havens works in topology under the supervision of Professor Inanc Baykur and is the recipient of our Five College Teaching Fellow Award. He has taught throughout the curriculum: calculus I, II, and III and linear algebra. His students find him to be an energetic, enthusiastic, and passionate teacher who can both challenge and inspire students. Due to his teaching success, he was selected as our Five College Teaching Fellow and taught at Amherst College in the fall.

Konstantinos Gourgoulias works in applied mathematics, applied probability, and high-performance computing under the supervision of Professors Markus Katsoulakis and Luc Rey-Bellet. Konstantinos has had a significant outreach impact on the data science communities on campus, in the Five Colleges, and beyond by being a co-founder and past co-chair of the “Graduate Researchers interested in Data” group (GRiD) and attending two conferences: the 2015 New York Finance Forum at Columbia University in the City of New York and the New England Statistical Symposium at UConn Storrs. The response was overwhelming! More than eighty people expressed interest, about thirty of them actively participating. It was gratifying to see that the Department of Mathematics and Statistics had a strong representation. The following graduate students in our department were registered participants: Betsy Camano and Emma Kearney from statistics and Rachel Gordon, Konstantinos Gourgoulias, Sandra Castro-Pearson, Ankita Shankhardar (who provided the photo for this article), and Oliver Spiro from applied mathematics. Other departments with a high attendance count were Biostatistics and Epidemiology, Computer Science, and several departments in the College of Engineering.

Starting at 6:00 pm on Friday, February 5, and continuing until noon on Sunday, February 7, students worked either in teams or on individually developed projects related to the PVTA data set. These projects were then presented to a panel of judges, Dr. Ios Orakosou from Hanover Insurance and Professor Amelia McNamara from Smith College. It was a challenge for the judges to select from the available projects, but in the end three stood out. The first was a study of the variance of headway; a natural quantity to study as it to lay the foundation of an event focusing on data from the Pioneer Valley Transit Authority (PVTA). During the period February 5–7, 2016 members of this group were given the opportunity to study a data set provided by PVTA, to pose their own questions, and then to answer them by using the data. After contacting PVTA management, they were supplied with more than a year of daily observations from one of their garages. Among other variables, the data included the GPS coordinates of the buses, the times of arrival and departure from the stops, and the number of passengers at any given moment.

Next, the GRiD group had to get the word out to students and researchers in data science at UMass Amherst and the other colleges in the area. The response was overwhelming! More than eighty people expressed interest, with about
is connected to the waiting time one has to spend at the bus stop. A different idea was to look at the number of people riding on a specific route during different times of day. By statistically calculating the peak times and routes, one could propose changes in PVTYA service so that the ridership is smoothed out. Finally, a team proposed using the data to infer where the buses were more likely to show mechanical or other problems that would cause them to be late.

Overall it was an extremely successful event that was strongly supported by the local data science community, various academic departments from UMass and the Five Colleges, and the Pioneer Valley Transit Authority. Martijn Theuwissen, and Josh Rickman, Manager of Operations and Planning at PVTYA, kick-started the event by explaining some of the finer points of the data set. For more information about the event, please visit http://gridclub.io/HackPVTYA.

GRiD, the group that sponsored the event, is an interdisciplinary graduate-student organization with members from all over the UMass Amherst campus. In the past, GRiD has organized numerous talks and a “HackEbola with Data” event in collaboration with Harvard Public Health, and has assisted with the ASA Five College DataFest. For more information about GRiD, you can visit http://gridclub.io.

Editors’ Note: The GRiD group that sponsored the event discussed in this article was co-founded by Konstantinos Gourgoulias. Konstantinos is a graduate student in applied math in our department. Because of his efforts with the GRiD group and other activities, he was awarded the Graduate Student Leadership Award. For details concerning this award, please see the previous article.

SPRING 2016 STATISTICS MASTER’S PROJECT SEMINAR

Nine graduate students took the Statistics Master’s Project course in the spring semester. Professor Erin Conlon organized the course and advised two different project groups; both groups worked on similar topics in data science.

Presentations were given by members of each group to the class approximately weekly, and the R programming language was the primary programming language used for the projects.

The first group included Doruk Cengiz, Osman Icoz, Yan Ling, and Matthew Schmidt, and the second group included Betsy Camano, Fei Fang, Mark Hagemann, Chenxi Wang, and Kun Wang. The first part of the project involved analyzing a real data set from Kaggle which is a platform for data-analysis competitions (https://www.kaggle.com). For this, both groups worked with the Titanic data set, which consisted of 891 passengers of the British ship that sank in the North Atlantic Ocean on April 15, 1912. The data set contained information for each passenger, including whether or not they survived the crossing, age, traveling class, gender, fare paid, and the number of other family members who were traveling with each passenger, among many other variables. The goal of the statistical analysis was to create statistical models that predict whether each passenger lived or died, based on the known variables.

Before statistical models were developed, students explored several data-analysis topics, including methods for data visualization and procedures for evaluating missing data. After these steps were completed, students developed several models to predict the binary outcome of survival. Several different approaches were used for creating models, including logistic regression and Lasso (least absolute shrinkage and selection operator). The results were compared using different metrics, including overall accuracy, sensitivity, and specificity, among many others.

The next part of the project focused on simulating data sets. For this, each group created synthetic data sets for the other group to analyze; the data consisted of a binary outcome with very high accuracy for most data sets. Many statistical and machine learning tools were used for the analyses, including logistic regression, Lasso, K-nearest neighbors, support vector machines, random forest, and gradient boosting. Students were able to predict the outcomes with very high accuracy for most data sets.

For the last part of the project, each group created an R package with functions related to the statistical analysis of the Titanic data set and the simulated data sets. An R package is an assembled set of R functions, their documentation, and examples. Each group also created a Shiny application for data visualization; Shiny is an online application structure for interactive web applications (http://shiny.rstudio.com).

ISOPERIMETRIC PROBLEMS

As told by Victor Blažić in his 2005 American Mathematical Monthly article, “The Evolution of the Isoperimetric Problem,” the isoperimetric problem has its origin in antiquity. According to the legend, Dido, the daughter of the king of Tyre, fled her home after her brother had killed her husband. After arriving on the north coast of Africa, she bargained to buy as much farmland as she could enclosure with an oxhide. She next proceeded to cut the oxhide into thin strips, but then she faced the problem of enclosing the largest possible farmland within the fixed total length L of the strips. Under the assumption that there were n strips of possibly different lengths and with total length L, Dido’s dilemma was to find the largest piece of land that she could enclose with these strips. Her dilemma is the first example of an isoperimetric problem; the first two syllables “iso” come from the Greek word meaning “equal.”

Most of us remember, from our calculus days, solving a special case of Dido’s isoperimetric problem. It is defined by choosing n = 4 and restricting the shape of the farmland to be a rectangle with sides of lengths x and y having total length or perimeter L = 2x + 2y and area A(x, y) = xy. In this rectangular setting Dido’s problem is to maximize the area function subject to the constraint that the perimeter of the rectangle is fixed to be L. Since L = 2x + 2y, if we solve for y and substitute into A(x, y) = xy, then the function becomes the one variable quadratic function

\[
A(x) = \frac{1}{2} x^2 - \frac{1}{2} L x + \frac{1}{2} L^2 - \frac{1}{2} L^2 - x^2.
\]

Taking derivatives, we obtain A'(x) = L - 2x, and so the value x = L/2 maximizes the area. But when x = L/4, we have y = L/4 - x = L/2, which means that the maximum area occurs for a square with sides of length L/2.

A nontrivial analysis shows that there is an analogous solution of a generalization of Dido’s problem to polygons with n sides.

Problem 1 (Dido’s Isoperimetric Problem for Polygons with n Sides) Given n a positive integer satisfying n ≥ 3, and \( R_n \) a polygon with n sides, then the polygon \( R_n \) that maximizes the area with fixed perimeter L = 2n is a regular polygon with sides of length 2n/L. The areas \( A_n \) of these regular polygons are strictly increasing functions of n as n tends to infinity and have the limiting value \( \pi \), which is the area of the disk enclosed by a circle with radius 1.

It is worthwhile to point out that when these regular polygons with perimeter 2n are centered at the origin in the plane \( \mathbb{R}^2 \), then as the number of sides n tends to infinity, these regular polygons converge geometrically to the unit circle, which is the boundary of the unit disk \( D \) of radius 1 centered at the origin. Therefore, if Dido had n strips of total length L = 2n, then her area-maximizing farmland would have the shape of a regular polygon with n sides of equal length and with total area strictly less than \( \pi n \) square units; the areas of these polygons converge to \( \pi n \) square units as n tends to infinity.

Our success in solving Dido’s isoperimetric problem for maximizing the area enclosed by a polygon with n sides and fixed perimeter inspires us to consider the following similarly posed perimeter-minimization problem. Does there exist a simple closed curve \( \Gamma \) in the plane of least total boundary length that encloses a domain \( \Delta \) of fixed area \( \pi \)? Domains that solve this problem are called isoperimetric domains. In a problem like this it is natural to restrict the regularity of \( \Gamma \) by adding the assumption that \( \Gamma \) is a differentiable curve.

The unit disk \( D \) appearing in the preceding paragraph is a solution of this problem. In addition, it can be shown that any such isoperimetric domain must be a translation of \( D \). Still more generally, if one allows the domain \( \Delta \) to have a finite number of differentiable boundary curves instead of just one, then the disk \( D \) is a domain of least total boundary length under the constraint that the area is fixed to be \( \pi \), and additionally, this isoperimetric domain is unique up to translations.

A proof that the disk \( D \) solves this general isoperimetric problem has a level of difficulty that far exceeds the previously considered cases where we restricted the shape of the domain to be a polygon with n sides. The natural theoretical tools for solving the isoperimetric problem in the plane can be found in the mathematical subject called geometric measure theory. Geometric measure theory also provides powerful techniques that generalize to other perimeter-minimizing problems in higher dimensions. For instance, this theory can be applied to prove that if \( \Delta \) is a differentiable domain in \( \mathbb{R}^2 \) with volume \( \frac{3}{4} \pi \), then the area of its boundary is at least equal to \( 4n \), which is the area of the boundary sphere of the ball \( B \) in \( \mathbb{R}^n \) centered at the origin and having radius 1. From multivariable calculus we also know that the volume of the ball \( B \) is \( \frac{4}{3} \pi \), and so balls of radius 1 are solutions of the isoperimetric problem in \( \mathbb{R}^2 \) with volume equal to \( \frac{4}{3} \pi \).

In other words, isoperimetric domains of volume \( \frac{4}{3} \pi \) in \( \mathbb{R}^3 \) are round balls.
Consider a polygon $\Gamma$ that encloses a domain where areas and boundary lengths may both be infinite.

In 1887 Lord Kelvin proposed an innovative generalization of classical isoperimetric problems, stated in the following question. The study of this generalization involves a number of deep mathematical insights.

Problem 2 (Kelvin’s Isoperimetric Problem in $\mathbb{R}^3$) How can three-dimensional Euclidean space $\mathbb{R}^3$ be partitioned into cells of volume 1 having the least amount of surface area between them?

Kelvin conjectured that the solution corresponds to the modified honeycomb shown on the left in Figure 1. To understand Kelvin’s isoperimetric problem, we first consider the more easily conceptualized version of Kelvin’s isoperimetric problem in two-dimensional Euclidean space $\mathbb{R}^2$, where the problem has been solved. One of the difficulties is that one must generalize the perimeter-minimizing problems considered earlier in this article to situations where areas and boundary lengths may both be infinite. Consider a polygon $\Gamma$ that encloses a domain $D(\Gamma)$ of area 1 in the plane $\mathbb{R}^2$ and suppose that we can tile or partition $\mathbb{R}^2$ by using copies of $D$. Let $\Gamma$ be a differentiable, simple closed curve in $\mathbb{R}^2$ having area 1 — each copy having been translated and possibly rotated — in such a way that the ratio of the perimeter to the area is the smallest in the following sense. If $\Gamma$ is the minimizing solution and $T$ is another tiling, then as $\Gamma$ tends to infinity, the ratio of the total length $L(\Gamma, T)$ of the boundary segments of the tiling $T$ inside the circle of radius $\Gamma$ centered at the origin and the total length $L(\Gamma, T')$ of the boundary segments of the tiling $T'$ inside the same circle converges to a number less than or equal to 1; in symbols

$$\lim_{\Gamma \to \infty} \frac{L(\Gamma, T)}{L(\Gamma, T')} \leq 1.$$ 

It turns out that the hexagonal tiling $H$ solves this isoperimetric problem in $\mathbb{R}^2$. In particular, if one tiles $\mathbb{R}^2$ by squares having sides of length 1 and thus having area 1, then this tiling uses more perimeter or boundary length on the average than does the pattern $H$. This isoperimetric-type result partly explains why honey bees make hexagonal prism combs rather than rectangular prism combs to store their honey. The reason is that the hexagonal prism combs store more honey with the same amount of bee wax.

We now return to Kelvin’s isoperimetric problem in $\mathbb{R}^3$, which is stated in Problem 2. The modified honeycomb has been ruled out as the solution of this problem. Indeed, in 1993 Denis Weaire and Robert Phelan discovered the structure shown on the right of Figure 2. Called the Weaire–Phelan structure, it has about 0.3% less surface area per unit of volume than the modified honeycomb. It is unknown whether the Weaire–Phelan structure is the solution of Kelvin’s problem in $\mathbb{R}^3$. We now turn to a different class of area-minimizing problems, which includes the following classical and fully understood special case.

Problem 3 (Plateau’s Problem) Let $\Gamma$ be a differentiable, simple closed curve in $\mathbb{R}^2$. Does there exist a differentiable surface $\Sigma$ in $\mathbb{R}^3$ of least area with boundary $\Gamma$?

Plateau’s problem is named after the 19th century Belgian physicist, Joseph Plateau, who studied it in a somewhat different physical context. Plateau realized that solutions of this least-area problem are in fact solutions of the following related, least-energy problem. Does there exist a surface $\Sigma$ of least energy that has boundary $\Gamma$? To understand this equivalence, Plateau showed that the surface tension of a soap film causes it to form a shape that locally minimizes the physical energy of the soap film, which in turn can be shown to be proportional to the area of the soap film. In particular, surfaces $\Sigma$ with boundary $\Gamma$ of least area can be realized as surfaces of idealized soap film with their boundary curve being replaced by a thin wire corresponding to the curve $\Gamma$. Therefore, one can perform the following experiment in order to possibly find a given solution of Plateau’s problem. Represent $\Gamma$ as a closed wire, and then dip it into a solution of soap and water in a bucket. If the conditions are right, then when the wire is pulled out of the soapy water solution, the soap film that forms on it is the physical realization of a surface of least area. This area-minimizing property of the soap film always happens if $\Gamma$ is the boundary of a unique soap film.

One issue that can arise in this physical approach to solving Plateau’s problem is that the soap film that forms may not be unique or might be only “physically stable.” In other words, the soap film might only represent a surface that is a local minimum of the area function rather than an absolute minimum.

Physically stable soap film surfaces $\Sigma$ bounding a contour $\Gamma$ have the property that sufficiently close or nearby surfaces with boundary $\Gamma$ have area greater than $\Sigma$, in this sense soap-film surfaces represent local minima of the area functional. It is not known if an infinite number of different idealized soap films can form with their boundary curves being some fixed differentiable wire contour $\Gamma$; it is conjectured that only a finite number of soap film surfaces can occur for such a $\Gamma$.

Plateau’s problem motivates the next definition of a minimal surface.

Definition 4 A differentiable surface $\Sigma$ in $\mathbb{R}^3$ is said to be a minimal surface if every point on the surface lies in a small subsurface of $\Sigma$ that has least area relative to its boundary; in other words, this subsurface is a solution of Plateau’s problem for its boundary.

An important unsolved research problem in the theory of minimal surfaces of finite area is a conjectured sharp estimate for the area of the minimal surface in terms of the square of the length of its boundary. More precisely, this conjecture states that if $\Sigma$ is a differentiable minimal surface in $\mathbb{R}^3$ with finite area $A$ and boundary of length $L$, then $A$ and $L$ satisfy the sharp isoperimetric inequality

$$4\pi A \leq L^2,$$

with equality if and only if $\Sigma$ is a round disk in $\mathbb{R}^3$.

In the case when $\Sigma$ is a domain in $\mathbb{R}^3$, then this inequality follows from, and is equivalent to, our earlier discussion that a round disk of radius 1 in the plane uniquely solves the isoperimetric problem for domains with area $\pi$; in this case, $L = 2\pi$ and so the estimate is sharp. In 1984 Peter Li, Richard Schoen, and Shing-Tung Yau proved that the sharp isoperimetric inequality stated in the last display holds if the minimal surface has at most two boundary curve components; in particular, it holds if the surface $\Sigma$ has one simple closed curve for its boundary.
ALUMNUS PROFILE: ROBERT POLLACK ’54

Bob Pollack’s multifaceted and productive career as an actuary, as a founder and president of a major insurance company, and as an investment advisor in private practice is a testimony to his determination, self-confidence, and hard work. Bob grew up in Holyoke, historically a city of working-class immigrants and the business-owners who employed them, located less than 15 miles from the college town of Amherst. His father, Harry Pollack, owned a grocery store there. As Bob recalls, his father woke up at 4:00 AM every workday and drove to Springfield to get the best available fresh fruit and vegetables for his customers. Harry was also Bob’s first teacher. “My father was fascinated by numbers, and he passed on this fascination to me.”

Bob discovered that he was good at mathematics, and so when he began his studies at UMass Amherst in 1950, it was natural that he majored in math. The building in which the Department of Mathematics was located in the early 1950s looked much different from the soaring, white Lederle Graduate Research Tower, in which the Department of Mathematics was located in 1950, it was natural that he majored in math. The current building, the Math Department building, was completed in 1965. The Math Department building is named in honor of the late Harry Pollack, who was a long-time Department of Mathematics and Statistics faculty member.

When Bob looks back more than six decades to his own undergraduate years, he is struck by the many changes that have transformed UMass Amherst from a campus of about 3,000 students to the major research institution it is today. “Today it is known as Mass Aggie, then Mass State until about 1947 or 1948,” he mentioned. “About that time it was renamed University of Massachusetts. My wife and I are pleased that we have been able to contribute to the vibrancy of this institution in our own small way. The quality of the actuarial program in the Department of Mathematics and Statistics is a real source of pride for us.”

Bob and Lynne have every reason to be proud, and our department has every reason to express our deep gratitude. Although the program has grown in size and stature, its essence remains the same. The Faculty and staff of the Department of Mathematics and Statistics provide a supportive and challenging academic environment that prepares our students for success. We are proud to have supported Bob’s academic journey and continue to support his ongoing involvement in the actuarial profession.

Editors’ Note: An article on the actuarial program, titled “Actuarial Science: Courses and Careers,” appears on pages 22–25 in the department newsletter for the 2014–2015 year. In it the five members of the advisory board for the actuarial program, all recent graduates, talk about themselves and their career paths as actuaries. This newsletter is available at http://www.math.umass.edu/files/newsletters/6330_final.pdf.

Making a Case for "Giving Back" to UMass Amherst

My name is James Francis, and I am a 1986 graduate from the Department of Mathematics and Statistics. In addition to contributing my ongoing support to the University, I happily represent the Department of Mathematics and Statistics on the College of Natural Sciences Advisory Board. At one of our recent board meetings, I presented an idea that I thought might inspire UMass alumni to make generous contributions to their alma mater. At the time, I used back-of-the-envelope calculations to demonstrate how, through compounding, the cost savings of a state university education relative to a private university education grow through time. The idea made sense to our group, and I was asked to write a more formal paper, which can be accessed at [http://www.math.umass.edu/sites/www.math.umass.edu/files/file-attachments/james_francl_paper.pdf](http://www.math.umass.edu/sites/www.math.umass.edu/files/file-attachments/james_francl_paper.pdf).

As current and former UMass students, I trust that we all understand the value of our education and how much we have saved annually in tuition and expenses relative to those who attend private schools. But what tends to get overlooked is just how dramatic those savings are when we view them over the years after we graduate. The impetus for writing this paper was no accident. At the time I originally got the idea, my two sons were in their senior year of high school and applying to colleges. I felt that it was incumbent upon me as a responsible parent to ensure that they understood the economic consequences of their decision at this stage of their lives. I wanted to convey not only the exceptional quality of a public school education and all that goes along with it, but also the obvious monetary value it provides, relative to a private institution.

I hope that you enjoy reading this paper and that it not only validates your decision to attend UMass, but also provides some guidance as to how much support you can pledge and still be ahead of the game. Thank you.

Editors’ Note: We would like to thank James Francis for his generous, ongoing support of the Department of Mathematics and Statistics and, in particular, the Jacob-Cohen-Killam Math Competition. James is a generous benefactor and a true friend of UMass Amherst. We are grateful to him for sharing his ideas about giving back, an idea that explains to alumni how they might express their gratitude to the Department and the University.

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NEW CHAPTER OF THE ASSOCIATION FOR WOMEN IN MATHEMATICS AT UMASS AMHERST

During January 2016 students in the department started a chapter of the Association for Women in Mathematics. The purpose of the Association for Women in Mathematics (AWM) is to encourage women and girls to study and to have active careers in the mathematical sciences and to promote equal opportunity and the equal treatment of women and girls in the mathematical sciences. The AWM website is located here: https://sites.google.com/site/awmmath/home.

The chapter was launched by sophomore Shelby Cox, who is a double major in mathematics and linguistics, with Professor Eric Sommers serving as faculty advisor. The students worked quickly to draw up chapter bylaws and elect officers. Twenty women, both undergraduate and graduate, attended the first meeting. At that meeting candidates for offices spoke eloquently about being women in the sciences at UMass Amherst and about their vision for building up a community to support each other and other women thinking of entering the math major and related STEM majors. Candidates had many exciting ideas for the chapter, including outreach to high-school and middle-school girls, a brown bag lunch series with faculty speakers, professional development activities, and homework help sessions for fellow students. The offices were highly contested, and the speeches were impressive. An electronic ballot was held, and the elected officers of the new chapter are Shelby Cox (President); Kayley Miskis, a junior (Vice President); Tori Day, a PhD student (Secretary); and Athena Higgins, a sophomore (Treasurer). Jennifer Li, a PhD student, will maintain the chapter webpage.

Women math majors in our department are 33% of all math majors. While this is well above the percentage of women majoring in engineering or computer science, it is considerably below the national average of 42% across all mathematics and statistics departments. We hope that the new AWM chapter will be a positive force in drawing and retaining women to study the mathematical sciences at UMass.