

# Mathematics & Statistics NEWSLETTER

University of Massachusetts Amherst

Academic Year 2013-2014 Volume 29

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AMHERST

## OPIE'S GEOMETRY RESEARCH HONORED WITH CHURCHILL FELLOWSHIP

Late last September, as **Morgan Opie** was beginning her senior year as a member of Commonwealth Honors College, she posted her first research paper *Extremal divisors on moduli spaces of rational curves with marked points* on the arXiv, an internet repository for scientific preprints. Opie, a double major in mathematics and physics, based the paper on her summer Research Experience for Undergraduates (REU) here in the UMass Amherst math department with Professor **Jenia Tevelev**.



Morgan Opie with Jenia Tevelev at the 2014 Undergraduate Awards Dinner

As a result of this research and her excellent academic record, Opie was invited to conferences, accepted at prestigious graduate schools – including Harvard, Princeton and Yale – and honored with a UMass Rising Researcher award. She was also named runner-up for the 2014 Alice T. Shafer Prize, a competitive national award for undergraduate women who excel in mathematics. Perhaps most impressive, Opie is now one of 14

Churchill Fellows nationwide: this honor comes with a scholarship worth about \$50,000 that will fund a year of postgraduate study at Cambridge University in England before Opie begins graduate school at MIT.

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## MIRKOVIĆ NAMED SIMONS FELLOW

Professor **Ivan Mirković** has been awarded a Simons Fellowship in Mathematics. The \$111,041 award for a sabbatical period between September 1, 2014, and August 31, 2015, includes funding to visit Harvard University, the University of Chicago, and Université Blaise Pascal in Clermont-Ferrand, France.



Mirković works in geometric representation theory, a field created primarily for the needs of number theory, and the Langlands Program in particular. Over time, it has acquired a central position

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## OBLOMKOV WINS NSF CAREER GRANT

Professor **Alexei Oblomkov** has been awarded a prestigious Faculty Early Career Development Program (CAREER) grant from the National Science Foundation. His proposal, “Knot invariants, moduli spaces of sheaves and representation theory,” was funded for \$420,000 over a period of 5 years. In this highly competitive program, only about 40 proposals in mathematics are funded each year, with fewer than half of those coming from pure mathematics.

Besides supporting Oblomkov’s research program, the CAREER award includes support for broadening our department’s undergraduate research (REU) program. The grant will support 7 undergraduate students for 2 months each summer over the course of the award. Current plans include building

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## KEVREKIDIS 2014-2015 ULAM DISTINGUISHED SCHOLAR AT LANL

Professor Panos Kevrekidis has recently been appointed the Stanislaw M. Ulam Distinguished Scholar for 2014-2015 at the Los Alamos National Laboratory (LANL). This is an annual award that enables a noted scientist to spend a year carrying out research at the LANL Center for Nonlinear Studies. The scholarship honors the memory of the brilliant Polish-American mathematician Stan Ulam, who was among the founders of what has now become ‘nonlinear science.’ A number of Ulam Scholars from 1985 until the present have made significant contributions to Laboratory efforts in nonlinear science, and many continue to collaborate with researchers in the technical divisions at LANL.

# MATHEMATICS & STATISTICS NEWSLETTER

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The Department of Mathematics and Statistics publishes its annual newsletter for alumni and friends.

You are important, and we want to hear from you! Please contact us at [dept@math.umass.edu](mailto:dept@math.umass.edu) to share your news, let us know how you are doing, and learn ways to become involved with the Department. Our website is a valuable resource for current happenings and news, so we encourage you to visit us regularly at [www.math.umass.edu](http://www.math.umass.edu).

Back Cover Image by Nicholas Schmitt, PhD'93: a genus 2 surface of constant mean curvature embedded in the 3-sphere, stereographically projected to 3-space and sliced open along a symmetry plane.

## FORMER HEAD'S FAREWELL

Friends, Colleagues, Alumni:

Among the many events this past academic year, a few deserve special mention. Graduate student **Nico Aiello** won UMass's Distinguished Teaching Assistant Award, a high honor for which he was nominated by his students. Our students' teaching is also valued by the Five College consortium which will inaugurate this fall the Five College Teaching Fellowship in Mathematics: a graduate student will be chosen each year to teach at one of the consortium's liberal arts colleges. Nico's award and the Five College Fellowship add to other recent teaching awards won by our faculty, including last year: **Panos Kevrekidis** (CNS Outstanding Teacher) and **Catherine Benincasa** (UMass Distinguished Teaching).

The department conducted three faculty searches, in analysis, probability, and statistics. All three searches identified excellent candidates, who were also deemed excellent by other universities. We are pleased that our analysis candidate, **Nestor Guillen**, accepted our offer and is joining our department this fall. We plan to continue the probability and statistics searches in the coming year.

Several of our faculty were recognized for academic excellence. **Alexei Oblomkov** won an NSF CAREER award. NSF describes the award as "a Foundation-wide activity that offers the National Science Foundation's most prestigious awards in support of junior faculty who exemplify the role of teacher-scholars through outstanding research, excellent education and the integration of education and research within the context of the mission of their organizations." **Andrea Nahmod** received a Simons Fellowship which, according to the Simons Foundation, "provide[s] funds to faculty for up to a semester long research leave from classroom teaching and administrative obligations." Nahmod used her fellowship to extend a sabbatical leave from one semester to two, which she spent with collaborators at MIT. **Ivan Mirković** will have a Simons Fellowship this year. Panos Kevrekidis was chosen by Los Alamos National Laboratory to be the 2014-2015 Stanislaw Ulam Scholar; he will spend the year at LANL.

My three-year term as Department Head ended this summer. It was an honor to be entrusted with the position and I thank you sincerely.

– *Michael Lavine*

## DEPARTMENT HEAD'S MESSAGE

It is with pleasure that I invite you to peruse this year's Newsletter, which will highlight just some of our faculty and students and their numerous activities in and around the Department.

Before giving a quick rundown of staffing changes in the Department, a few words of appreciation are in order. First I would like to thank **Michael Lavine** who served the Department for three years as Head. When I assumed the duties of Head on July 1, having served as Associate Head for the previous three years, I knew that I could rely on an exceptional group of staff members, led by **Iлона Trousdale**, to help me learn the job and maintain a smooth schedule of administrative tasks.



All the research and teaching that takes place every year is facilitated by the work of the staff, so I would like to take this chance to thank them for the dedication they collectively bring to their domains. The Department said goodbye last year to **Christine Richotte** (who served the Department as Graduate and Undergraduate Programs Assistant for several decades), **Kim Stone** (who was the Assistant to several Heads and Associate Heads) and **Joy McKenzie** (who ran the copy room for many years). We are extremely pleased to welcome **Jacob Lagerstrom** and **Christine Ingraham** who joined us last year, replacing Chris and Kim, respectively; they have quickly become highly valued members of the staff. More recently, **Lisa Bergman**, **Cathy Russell**, and **Kam Kit Wong** joined **Sarah Willor** and **Carla Mokrzecki** in the business office led by **Phil Szczepanski**. **John Folliard** joined **Ken Pollard** and **Alan Boulanger** in the Research Computing Facility this past spring.

I also have the support of a very strong group of faculty members filling leadership positions. **Tom Braden** and **Siman Wong** continue in the role of Graduate and Undergraduate Program Director, respectively, and **John Staudenmayer** continues as Chief Undergraduate Advisor. I'm very pleased that **Mike Sullivan** and **Robin Young** have agreed to join the administrative team as Associate Head and Associate Head for Curriculum, respectively. The latter position is a new one for our department, reflecting our commitment to increase the coordination of our curriculum with other departments within the College of Natural Sciences.

In Academic Year 2014-15, we have a fairly large influx of new Visiting Assistant Professors in the Department, and can in fact report an increase by one for the full slate of VAPs. Joining us this fall are **Ravi Kalpathy** (statistics), **Nathan Sunukjian** (topology), **Jun Wen** (number theory), **Nathan Totz** (analysis), and in January 2015, **Yaping Yang** (algebraic geometry/representation theory). Increasing the number of VAPs is one of my priorities for the next three years. We are also extremely pleased to have several postdocs funded by federal grants. They are: **Ioannis Pantazis** and **Georgios Arampatzis** funded by the Department of Energy through a grant garnered by **Markos Katsoulakis** and **Luc Rey-Bellet**; **Simon Thalabard**, funded by an NSF grant awarded to **Bruce Turkington**; as well as **Stathis Charalampidis** and **Bettina Gertjerenken** working with **Panos Kevrekidis** through a combination of grants from the European Research Council and the Air Force. **Andrew Bray** is a postdoc funded by Michael Lavine through a Five College Statistics grant.

The Strategic Planning process begun by Chancellor **Kumble Subbaswamy** two years ago has now moved to the level of departments and colleges. We are taking this opportunity to write a short but descriptive "Departmental Scan" across all areas and propose short term as well as programmatic enhancements to the undergraduate program. In the spring semester, we will develop a more detailed set of strategic goals for the future. We are pleased to have an updated website, which will allow for several important enhancements over the coming year. The Department is also excited to be working to plan for future space reconfigurations. We will coordinate with Campus Planning to design common spaces for specific uses. In particular, the Department hopes that, as a new Physical Sciences Building is scheduled to come online in about 5 years, a restructuring of space in the Lederle Tower will allocate significant square-footage to our footprint in the building. This comes at a very good time, since the Department's strategic plan calls for significant growth in all areas, including graduate students and postdocs/VAPs over that time period.

– *Farshid Hajir*

## 2014 APPLIED MATH M.SC. PROJECTS: FROM ATOMIC PHYSICS AND MATERIALS SCIENCE TO FINANCIAL MATHEMATICS AND BEYOND

This year the Applied Mathematics Masters' Program tackled a diverse variety of themes in the context of its yearly project. In particular, a team of 7 students, consisting of **Matthew Brown, Cassandra DePietro, Domonic Mei, Christopher Rocheleau, Shaina Rogstad, Cortney Tilley** and **Wenlong Wang** were immersed in the challenges of the theme of *Bose-Einstein condensates in atomic physics*, as well as of that of *granular crystals in materials science*, while **Orhan Akal** and **Jan Henry** examined deterministic (and potentially chaotic) models of supply-demand-pricing and inflation in financial markets. All three topics were led by Professor **Panayotis Kevrekidis** during the year long effort.

The phenomenon of Bose-Einstein condensation, which constituted the first theme of this year's studies, is a phase transition originally predicted by Bose and Einstein in 1924. In particular, they showed that below a critical temperature  $T_c$ , a finite fraction of particles of a (integer spin) boson gas condenses into the same quantum state, known as the *Bose-Einstein condensate* (BEC). As a prototypical Bose gas, think of your favorite gas of alkali atoms (e.g.  $^{87}\text{Rb}$  or  $^{23}\text{Na}$  or  $^7\text{Li}$ ), and as relevant critical temperatures, think of the lowest temperatures in our universe ranging at a few hundreds of nK (nanoKelvins= $10^{-9}\text{K}$ ). Although Bose-Einstein condensation is known to be a fundamental phenomenon, connected to superfluidity and superconductivity, BECs were only experimentally realized 70 years later: this major achievement took place in 1995 and has already been recognized through the 2001 Nobel Prize in Physics.

One natural question regarding BECs, especially from the point of view of applications, is what they may be useful for. Perhaps the simplest and most important aspect of the answer is that they are a prototypical and extremely "clean" experimentally tractable system, which can operate as a vehicle for examining numerous fundamental phenomena of quantum and wave physics. In addition, there are numerous other areas of potential BEC applications. For instance, BECs have been experimentally demonstrated to be usable for performing remarkable tasks such as the implementation of the Gaussian sum algorithm for factoring numbers, by exploiting higher order quantum momentum states, improving in this way the algorithm's accuracy beyond its classical implementation. This is in line with the development of the Shor algorithm as an efficient quantum mechanical way

to factorize large numbers, a task thought to be classically intractable. An additional very interesting proposal that has been made is that of using BECs, in the presence of periodic optical lattice potentials, as quantum computers that can perform logic operations with so-called qubits. In optical lattices also, atomic analogs of semiconductor electronic circuits (the so-called "atomtronics") have been proposed, in order to realize quantum devices such as diodes and transistors. Furthermore, while the principal difficulty in using BECs in everyday applications has been their ultra-low temperatures, there have recently been numerous significant steps towards overcoming such a difficulty, including the production of BECs even in prototypical bosons in the form of photons in recent state-of-the-art experiments which occurred at *room temperature*. This suggests that BECs are, arguably, far closer to applications than they were a few years ago and an enhanced understanding of their properties and nonlinear dynamics is critical, especially in light of the above applications.

The dynamics of BECs can be theoretically described by the effective mean-field, Gross-Pitaevskii (GP) equation. This is a variant of the famous nonlinear Schrödinger (NLS) equation, incorporating an external confining potential  $V_{\text{ext}}$ , of the form:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + g|\Psi|^2\Psi + V_{\text{ext}}(\mathbf{r})\Psi \quad (1)$$

where  $\Psi = \Psi(x,y,z,t)$  is the BEC wavefunction (the atomic density is proportional to  $|\Psi(x,t)|^2$ ),  $\Delta$  is the Laplacian,  $m$  is the atomic mass and  $g$  is proportional to the atomic scattering length ( $g > 0$  for Rb and Na, while  $g < 0$  for Li atoms). The NLS is a universal model describing the evolution of complex field envelopes in non-linear dispersive media. As such, it appears in a variety of physical contexts, ranging from optics to fluid dynamics and plasma physics, while it has also attracted a strong mathematical interest.

The nonlinearity in the GP (NLS) model allows the prediction, and description of important and experimentally relevant nonlinear effects and nonlinear waves, such as solitons and vortices. These so-called matter-wave solitons and vortices can be viewed as fundamental non-linear excitations of BECs, and as such have attracted considerable attention. Furthermore, they have also been observed in many elegant experiments. It is exactly these states that the M.Sc. team tackled. The students used techniques that they had learned in numerical analysis (including fixed point iterations, numerical linearization and eigenvalue computations, and ODE as well as PDE integration

techniques and schemes) in order to be able to provide a set of computational tools for examining dark solitons in one-dimensional BECs and vortices in two-dimensional settings. They complemented this computer-based analysis with theoretical tools from the near-linear theory of partial differential equations (for small amplitude solutions), as well as with a highly non-linear analysis where these solitary

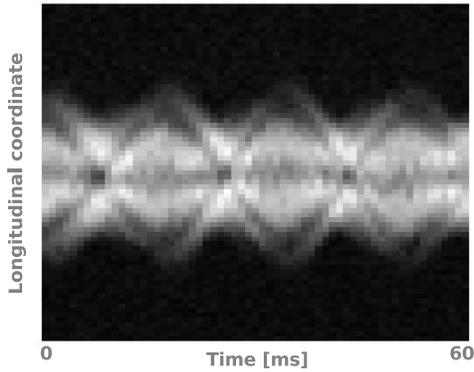


FIG. 1: The top panel shows the space-time evolution of an experiment in the group of Prof. M. Oberthaler (Heidelberg, Germany) involving 3 oscillating and interacting dark solitary waves (one quiescent in the middle and two symmetrically located ones impinging upon it). The contour plot corresponds to the atomic density (so there are atoms where the contour is white and atoms are absent where it is dark). Using a similar grayscale code, the bottom panel shows a series of snapshots (about 25 ms apart) from the lab experiments of Prof. D.S. Hall at Amherst College showing two oppositely charged vortices (a so-called vortex dipole) located at an equilibrium position. The last snapshot is after expansion at the end of the experiment, hence it is spatially wider.



waves were considered as individual “particles” moving inside the BECs. In the latter analysis, the original NLS was mapped into a set of ordinary differential equations for the motion of the individual wave-particles, which were subsequently analyzed by means of dynamical systems techniques. This way the students integrated in their year-long project the technology they had been taught in their graduate courses, including the numerical analysis sequence (Math 651-652), Ordinary Differential Equations (Math 645 and 532), and Partial Differential Equations (Math 731 and 534).

The result of the team’s efforts was an elegant analysis motivated by recent experiments in the group of Prof.

M. Oberthaler in Heidelberg, Germany where multiple oscillating and interacting dark solitons were observed, as well as a detailed understanding of the experiments conducted nearby, at Amherst College in the group of Prof. D. Hall. In fact, in an interesting twist, the team took a field trip to visit Prof. Hall’s lab and got a first hand impression of how the BECs arise in such an ultra-cool (literally!) lab. Figure 1 contains a sampler of the type of experimental phenomenology that the group was able to handle and fundamentally explain on the basis of the individual motions and pairwise interactions of solitons and vortices.

The second project that the M.Sc. students tackled involved the dynamics of granular crystals, a theme of emerging interest in the realm of materials science. The crystals consist of chains of beads assembled either in one-dimensional (most commonly) or two dimensional (e.g. hexagonal) configurations, held together by rods, whereby the beads are interacting via the so-called Hertzian contacts, namely a power law nonlinear interaction between nearest neighbors. More generally, these crystals can be thought of as networks of elastically interacting particles consisting of various materials and geometries. From an engineering perspective, their appeal lies in tunability of their dynamic response, making them relevant for a host of applications including shock and energy absorbing layers, actuating devices, acoustic lenses, acoustic diodes and sound scramblers. From a mathematical and computational perspective, granular crystals provide an ideal testbed for validating

novel principles of mathematical physics, at the interface and through the interplay of nonlinearity and periodicity or heterogeneity/disorder.

The dynamics of interacting granular crystals is described by the model

$$M_n \ddot{u}_n = \sum_{j \in I} F_{j,n}(u_j - u_n) \quad (2)$$

where  $n \in \mathbb{Z}^d$  is the lattice index with  $d = 1$  corresponding to a 1D chain and  $d=2$  to a 2D lattice.  $M_n$  is the bead mass,  $I$  represents the indices of neighboring particles that are in contact with bead  $n$ , the force function  $F_{j,n}$  describes the interaction of bead  $n$  with its neighbor  $j$ , and  $u_n(t) \in \mathbb{R}^d$  represents the displacement of bead  $n$  from its equilibrium

position at time  $t \in \mathbb{R}$ . In the 1D chain there is only interaction with the left and right neighbor, and thus Eq. (2) reduces to,

$$M_n \ddot{u}_n = F_{n-1,n}(u_{n-1} - u_n) - F_{n,n+1}(u_n - u_{n+1}) \quad (3)$$

where the force functions are given by the contact law  $F_{n-1,n}(x) := A_{n-1,n}[\delta_{0,n} + x]_+^p$  where  $A_{n-1,n}$  is the material parameter related to the contact of bead  $n - 1$  and  $n$ , and the bracket is defined by  $[x]_+ = \max(0, x)$ ; that is, there is no tensile force. The precompression term  $\delta_{0,n}$  is an equilibrium displacement induced by a static load  $F_0 = A_{n-1,n} \delta_{0,n}^p$ , which controls the strength of the nonlinearity, ranging from purely nonlinear (zero precompression) to almost linear (large precompression). For the special Hertzian case of elastic grain interactions,  $p = 3/2$ , but the considerations can be generalized to arbitrary  $p$ . Equation (2) is an idealized model which ignores effects such as disorder and rotational dynamics although it is also straightforward to include a dashpot to model dissipation. Despite the simplicity of (2), it has been shown to provide a reasonable description of the dynamics of granular crystals, even in 2D settings.

Here, the M.Sc. students tackled some of the most prototypical excitations that such a system can support. In particular, in the 1d context, they explored how a boundary excitation could spontaneously lead to the formation of traveling waves and analyzed through theoretical (based on Fourier transforms and co-traveling frames) and numerical techniques such solutions and their spatial decay properties. Another large class of solutions that was analyzed in comparison/contrast to the linear wave equation (arising for  $p = 1$  above) was that of shock waves and the phenomenon of wave breaking was observed. Not only were these characteristics visualized and appreciated in the context of “monomer” chains of one type of beads but also in that of heterogeneous chains such as dimers with two distinct types of beads. An example of the exact type of traveling wave obtained is shown in Figure 2. Finally, in the presence of individual “defects” in the chain, the team was able to observe the well-known phenomenon of exponential localization at the defect and the formation of time-periodic states in the form of the so-called “discrete breathers”.

Finally, the third team (O. Akal and J. Henry), motivated by an effort to understand the recent economic recession, studied of earlier similar events, including the Great Depression and the New Deal implemented by President Roosevelt to address it through a series of Domestic Programs. The team then turned to the study of nonlinear

ordinary differential equation models of particular sectors of the economy, focusing in particular on the minimalist Walrus-Keynes-Phillips (WKP) model. In the latter, the aggregate supply and demand are connected to the actual inflation rate, as well as to the expected inflation rate through a series of three ordinary differential equations which are weakly nonlinear. An analysis of the system using the tools of dynamical systems for a range of parameters used in the bibliography revealed the potential not only for stable and unstable limit cycles and periodic behavior, but also for

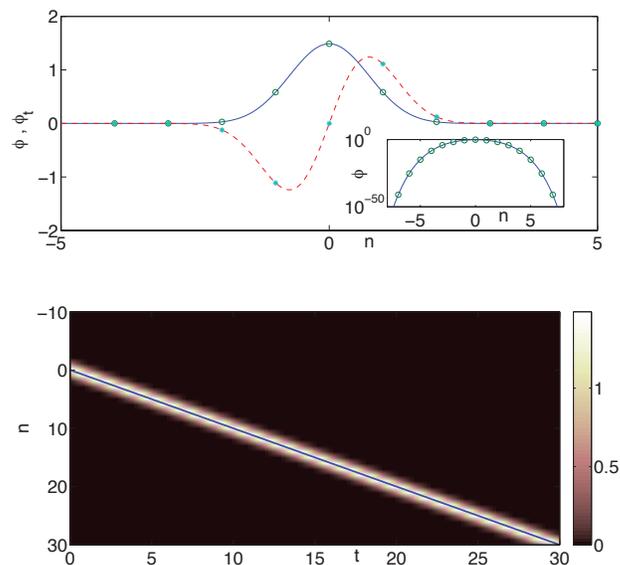


FIG. 2: The top panel shows an example of the numerically exact (up to a prescribed accuracy) traveling wave of a granular lattice (blue solid line), while its corresponding velocity profile is given by the red dashed line. The inset shows the spatial decay of the wave in a semilog plot. The evolution is indicated in the space-time plot of the bottom panel, clearly illustrating the constant speed propagation of the wave (indicated by the constant slope in the space-time evolution).

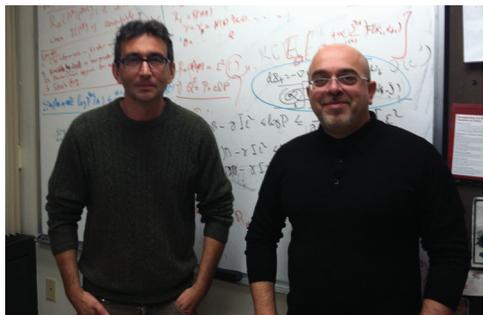
chaotic dynamics. Subsequently, the team tried to connect this behavior with another basic model that presents similar characteristics, namely the driven Van der Pol (VdP) model. The standard VdP model is a classical example featuring the formation and fast-slow system analysis of limit cycles, as it is taught in our advanced ordinary differential equation courses. However, the periodically driven variant thereof is an effective three-dimensional system and as such presents bifurcations from periodic to chaotic dynamics. The team analyzed these features and regimes of distinct behavior and then related this simpler model to the corresponding features of the WKP model. In this way, they provided a basic quantitative understanding of conditions under which

periodic and chaotic dynamics can emerge in supply-demand chains of different sectors of the economy.

The three teams assembled their conclusions in an hour long summary that was presented to the Department on May 1, 2014. The detailed analyses by the 5 speakers (Shaina Rogstad, Wenlong Wang, Matthew Brown, Domonic Mei and Orhan Akal) and the numerous movies highlighting the dynamics of the different systems were followed by a lively discussion which continued afterwards in a reception co-sponsored by the Department that celebrated the team's efforts. This marked the culmination of a successful year in the M.Sc. Program. Next year, Professor **Markos Katsoulakis** will undertake a series of projects under the general theme of "Uncertainty Quantification", a very timely and broad topic that is certain to offer numerous challenges and learning opportunities for the students that will participate in it, continuing the long and successful tradition of the Applied Mathematics M.Sc. Program Projects.

## QUANTIFYING UNCERTAINTY AND SENSITIVITY FOR COMPLEX SYSTEMS

Professors **Markos Katsoulakis** and **Luc Rey-Bellet**, with research partners at Brown University and the University of Delaware, will share a three-year, \$2.3 million grant from the U.S. Department of Energy to develop new methods to improve mathematical modeling of multi-scale, complex systems. New strategies are expected to have applications in energy research and materials synthesis.



Luc Rey-Bellet and Markos Katsoulakis

Predictive mathematical models and algorithms have long complemented theory and experiments in applied sciences and engineering, but such computational models are now more important than ever because of the increased complexity of the problems plus advances in computing

capabilities. The vast predictive potential of modeling and efficient simulation of complex systems is reflected in the 2013 Nobel Prize in Chemistry being awarded for the development of multi-scale models for complex chemical systems.

A primary potential practical application is to design high-efficiency, cost-effective bimetallic catalysts using relatively inexpensive metals, allowing storage and production of clean hydrogen fuel from readily available sources such as ammonia. Here the role of uncertainty and sensitivity quantification in this chemical process turns out to be crucial, because the design of bimetallic catalysts rests on understanding how sensitive the catalyst's performance depends on its parent metals. Experiments show that this depends on the micro-geometry of the arrangement of the two metals, that is, the structure and ordering of their layers. Given all the choices one has in selecting materials and geometries for these two-metal catalysts, this becomes a very complex system to model.

One of the key goals of the research team is being able to systematically evaluate which metal combinations in the catalyst are the most efficient and cost effective. The challenge is an example of a model where new mathematical and computational techniques for assessing uncertainty and quantifying sensitivity can be extremely productive.

Over the next three years, the multi-institution team will develop new mathematical tools that describe uncertainty and model sensitivity using information theory, probability theory, statistical methods such as model selection and model reduction, rare-events methods, multi-scale analysis and parameterization of coarse-grained models from finer scales and data.

Mathematical models are now routinely being asked to account for systems of increasing complexity, handling millions or even billions of variables. In addition, a model must integrate and account for data over different temporal and spatial scales from the molecular level all the way to the everyday macroscopic scale.

Interactions across scales are a unifying feature in all complex systems that we experience in everyday life. For example, consider the effect that a single vehicle breakdown during rush hour may have on overall traffic flow, even at very large distances from the scene. Taking all the different variables and mechanisms – vehicle speeds,

sizes, road network topology, weather, traffic volume and so on – into account, represents a typical complex, multi-scale, multi-physics modeling, simulation and analysis problem that challenges current applied mathematical methods.

Handling real-life systems with such unprecedented levels of complexity and multi-scale features requires not only more powerful computational capabilities, but also new mathematics. High-performance computing can allow simulation of at least some complex systems, but there are important concerns related to the effectiveness and reliability of the predictive computational models.

As in the traffic example above, all such models depend on a large number of mechanisms and parameters, but it is not immediately obvious which ones critically affect the final predictions and which can be ignored. Another closely related source of uncertainty is insufficient knowledge of a particular highly complex system.

Markos and Luc believe their research has great potential for wider impact in a number of fields because it will lay the mathematical foundations for uncertainty quantification and sensitivity analysis in a broad class of complex systems typically encountered in physico-chemical and biological processes, in atmosphere and ocean science, and in other types of complex networks.

## THE HISTORY OF MATHEMATICS RETURNS

Mathematics has a rich and fascinating history.

There are the apocryphal stories: Some familiar – the ancient Pythagoreans threatened capital punishment to guard their secret knowledge that  $\sqrt{2}$  is irrational. Some obscure – Einstein always erased his own chalkboards. And some (purporting to be) humorous – Siegel, lecturing in Göttingen and asked by a passing janitor if he'd noticed the lecture hall was empty, replied (in German) "Yes, but the lecture was wonderful!"

There are the well-documented accounts: Euler wrote beautiful letters to a German princess, teaching her everything from astronomy to Newton's *Principia*. Newton preferred to learn geometry from Descartes' *Analytic Geometry*, rather than from Euclid's *Elements* – a book reprinted more than any other, except the Bible, and for centuries studied religiously by geometry students – this may explain why Newton was able to invent calculus and

revolutionize the mathematics of his time. And there is Fibonacci. While almost everyone knows the sequence 0, 1, 1, 2, 3, 5, 8, 13, ... , that wasn't his most important contribution. His father, a wealthy Italian merchant, often sent his son on errands around the Mediterranean. Along the way, Fibonacci learned math from Islamic scholars, who at that time were the world's greatest mathematicians. Once he returned to Pisa in 1200, Fibonacci wrote *Liber Abaci*, the landmark arithmetic text that introduced Arabic numerals to the Western world.

Many mathematicians find their mathematical heritage – the passing of information from teacher to student in an unbroken chain over thousands of years – a source of great pride. Yet this is difficult for undergraduates to grasp without knowing that history. Indeed, most students are exposed to math in high school and introductory college courses as an immutable compilation of theorems and problem-solving tools. The history of how these ideas came to be – and why – is left by the wayside.

And so, last spring – for the first time in many years – the History of Mathematics (Math 475) was again taught in our department. "Mathematics is a beautiful, personal, and creative art," one student wrote in her Math 475 final examination paper. "How can one appreciate its wonders without understanding its early artists?" Indeed, how will our students make discoveries on their own without learning how math's greatest thinkers made theirs?

As part of the course, Professor Jenia Tevelev required each student to write a long essay based on primary sources. Students had to navigate not only the archaic language of the past, but also the reasoning and logic used in these historical texts. (This, by the way, is very much in the spirit of Siegel's approach to teaching and learning math.) Each was also required to give a 30-minute presentation on a different topic in math history, and then lead a class discussion about it. For many, this was their only opportunity to do so in a math course. The quizzes and exams in Math 475 were also different from those in other math classes. On one quiz, students were asked to consider whether the opening lines in the *Declaration of Independence* resemble a Euclid-style argument in the *Elements*. A question on the final exam even asked students to discuss the attributes of perspective drawing in Vermeer's painting, *The Music Lesson*.

By the end of the course, students were left with a better sense of the history of mathematics – and how they might help to make history themselves!

as a bridge between number theory, representation theory, algebraic geometry and, more recently, the physics of quantum field theory. Some of Ivan's results form the basic building blocks of the Geometric Langlands Program. His work on modular representation theory has been influential in a number of mathematical disciplines. The research program funded by the Simons award deals with extending the theory of loop Grassmannians, "critical level" representation theory, and the proposed construction of geometric foundations for arithmetic number theory.

Independently, Mirković has also been awarded a *Simons Visiting Professorship* and an *Eisenbud Professorship* for the program in geometric representation theory during the 2014 Fall semester (August 18 – December 19, 2014) at the Mathematical Sciences Research Institute in Berkeley, California. This MSRI program is based on current opportunities for consolidating geometric representation theory and quantum field theory that have opened up through the use of methods from derived algebraic geometry.

Mirković's fellowship was awarded by the Simons Foundation, the charitable institution founded in 1994 by James and Marilyn Simons. A mathematician who translated his



Jim Simons

expertise in pure mathematics into an extraordinarily successful tool in financial markets, Jim Simons is not only a well-regarded mathematician, but also a major hedge fund manager and prominent philanthropist.

Simons began his career in academia, teaching at Harvard and MIT, as well as working at the Institute for Defense

Analyses in Princeton; during the 1970s, he chaired and helped strengthen the mathematics department at SUNY Stony Brook. In mathematics he is best known for his work on minimal varieties and the high-dimensional Bernstein Problem, while in physics he is revered as co-inventor of the *Chern–Simons 3-form* which theoretical physicist Ed Witten recognized as the *action integrand* for a fundamental example of topological quantum field theory, now called the *Chern–Simons theory* of 3-dimensional manifolds.

In 1982, Simons started Renaissance Technologies, a private hedge fund that uses computer-based mathematical models to predict price changes in easily-traded financial instruments. These models are based on analyzing as much data as can be gathered, then looking for *non-random movements* to make predictions.

A recent *New York Times*\* profile ranked Simons the 93rd-wealthiest person on the planet; earlier, the *Financial Times* named him "the world's smartest billionaire." And according to Theodore Aronson, principal of a quantitative money management firm with \$29.3 billion in assets:

*There are just a few individuals*

*who have truly changed how we view the markets.*

*John Maynard Keynes is one of the few.*

*Warren Buffett is one of the few.*

*So is Jim Simons.*

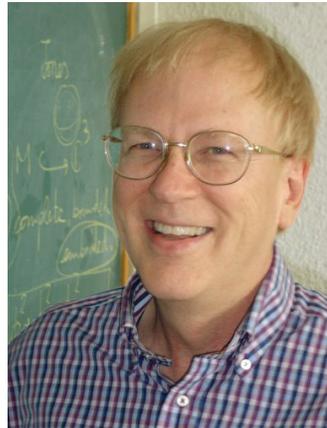
Simons's philanthropic activities include education, health and scientific research, particularly mathematics, physics, life sciences and autism research. He also contributes to the Democratic Party. He has made substantial financial commitments to several scientific institutions, including the Simons Center for Geometry and Physics at Stony Brook, which competes with the world's best institutions in mathematics and theoretical physics.

Since 2012, the Simons Foundation has been awarding the Simons Fellowships, which double the sabbatical leave of a recipient from one semester to a full academic year at full salary. Up to 40 Simons Fellowships are granted annually in mathematics; our department has benefited from one each year, with previous awards going to **Paul Gunnells** (2012-13) and **Andrea Nahmod** (2013-14).

\*Tuesday 8 July 2014: <http://www.nytimes.com/2014/07/08/science/a-billionaire-mathematicians-life-of-ferocious-curiosity.html?ref=science>

## FACULTY PROFILE: AN INTERVIEW WITH WILLIAM H. MEEKS III, THE G. D. BIRKHOFF PROFESSOR OF MATHEMATICS

The Birkhoff Professorship was created in the late 1960s as our department began its Ph.D. program and was developing a more research-oriented focus. It is named for one of America's preeminent mathematicians, George David Birkhoff, who made breakthroughs in geometry and dynamical systems, taught for 3 decades at Harvard, and whose 1926 Ph.D. student, the late Professor **Marshall H. Stone**, was the first holder of this position. Stone, a member of the National Academy of Sciences, was among the world's most prominent mathematicians of the mid-twentieth century, and he had a strong Amherst connection: his father, Harlan F. Stone, who served as the 12<sup>th</sup> Chief Justice of United States, had studied first at Mass Aggie and then – after being expelled from our Scientific Agriculture program for leading other students in pranks! – at Amherst College in the early 1890s. In 1986, several years following Marshall Stone's retirement, **William H. Meeks III** became the second holder of the Birkhoff Professorship.



Bill Meeks

**Bill Meeks** was already a world-renowned contributor to geometry and topology by the time he joined our faculty, so his appointment as Birkhoff Professor was a great leap forward for the Department's research program. With former colleagues **David Hoffman** and **Joel Spruck**, he founded the **Center for Geometry, Analysis, Numerics & Graphics (GANG)**. Bill and David were doing groundbreaking work on minimal surfaces using computer graphics at the time, and suddenly UMassAmherst was on its way to becoming a major center for geometric analysis. Soon other young and creative geometric analysts, like Professors **Markos Katsoulakis**, **Rob Kusner** and **Franz Pedit**, joined our department and GANG. And though the old GANG lab has morphed into the Department Common Room, even today – nearly 3 decades later – the GANG website [www.gang.umass.edu](http://www.gang.umass.edu) remains an internationally recognized source for minimal, constant mean curvature, and other special surface images (some of which grace this Newsletter).

The Newsletter interviewed Bill Meeks this summer. Here is some of what he had to share:

**Newsletter:** Bill, tell us about your early years and how you became interested in mathematics.

**Meeks:** I was born and brought up in Alexandria, Virginia. My early life and intellectual development were significantly influenced by my Dad, who had been classified as a “genius” on an IQ test he took when entering the military during World War II and as a result was appointed an officer. In the ensuing years after the war, my Dad became a successful businessman. We spent time together discussing investments, and I read some of the books he had purchased on how to become wealthy. These books were full of very imaginative strategies for becoming rich! Intrigued by these ideas, in my sophomore year of high school I took a part time job as a door-to-door salesperson. Getting good at this job in a short time, I found I could sell almost anything to anybody. The company recognized my talent and quickly made me their chief trainer of new recruits. Whenever I wanted to make a few hundred dollars, I would go out for a few hours to sell storm windows, aluminum siding or awnings – back in 1963 earning a few hundred dollars was like earning a thousand dollars in today's money. Making money with this job was easy, and soon working 8 hours a month was enough to buy all the frivolous things of youth.

I'll bet you won't be surprised that I did not do well in school! For example, I ended up failing my 8<sup>th</sup> grade science class. The main reason I failed was because I would tell my teacher daily that her scientific explanations and interpretations were incorrect; yet I never pointed out to her why what she told the class was wrong. In 11<sup>th</sup> grade I had this same teacher in a new class called Chem-Study. But this time around I understood that if you are going to say anyone is wrong in explaining something, you must have a better explanation to go along with your criticism. This chemistry class was a nationally promoted course that had been created as America's response to the successful Russian launch of its Sputnik satellite. [A note to younger readers: that rocket launch took place in Fall 1957, when Bill was 10.] The exams made by the Chem-Study Project were long and difficult. Because I already had a deep understanding of chemistry from self-study, the few problems which were marked incorrect on these SAT-like tests were not usually wrong at all – as I repeatedly explained to my teacher – but by now, even she agreed with me. After about a month of these after-exam

interactions, this same teacher who had failed me in 8<sup>th</sup>-grade Science took a very different view of me as a student. She overlooked my youthful enthusiasm, and – one day after class – asked me if I would be willing to transfer from the regular section into her honors chemistry section and to work as her lab assistant. Of course, I said “Yes!” After making the transfer, I even got the chance to teach the honors class when my teacher was out of the room. During this period of time, I was reading Dirac’s book on Quantum Mechanics. It was a time of intellectual encouragement and excitement for me.

While still in high school, my senior year experience in learning Calculus helped form my future approach to learning mathematics. On the weekend before my Calculus course began, I went to the public library, studied all the calculus books there, and learned about 30% of the material for my class. Perhaps “learned” is not the correct word; it was more like I became familiar with the main vocabulary, results and techniques used in Calculus. I have always had an excellent memory and it was easy to memorize the derivatives of all the classical functions, even not knowing exactly what they were. Having done this simple exercise, this ended up saving me vast amounts of time as the actual course progressed, and it gave me a tremendous edge in eventually learning the course material.

**Newsletter:** You have told us about your high school experiences, now tell us how these influenced your early college studies and what happened during those years.

**Meeks:** I was always quite “good” at mathematics in high school and in my first two years at college. But my love and aptitude then was theoretical physics. In my junior year at college I transferred to U. C. Berkeley because of their first rate Physics Department. Just before my first classes began at Berkeley, a random event occurred that profoundly changed the rest of my life. I was browsing for textbooks at Cal-Books in Berkeley and came across a small, thin text by Henri Cartan on the calculus of one complex variable. While reading the preface of the book, I just started laughing out loud because of the strange language that was coming across, phrases like “Rings of meromorphic functions on domains of the complex plane.” What were these strange objects that were completely new to me? Intrigued by the style of writing and the language in this book that seemed to be more like poetry than mathematics, on the spot I bought the book and went to the Registrar’s office to sign up for a theoretical math course, a course that is very close in approach to our Math 300 [the introduction to proofs class at UMassAmherst].

Another piece of pure luck was that my section of this “Math 300” class was being taught by a brilliant mathematician in number theory from the University of Nice in France, who was an amazing individual and teacher. His name was Michel Mendes France – and incidentally his father had been the Premier of France in the 1950s. Besides having a deep insight into the nature of mathematics, which helped in his explanations, my instructor also shared with us stories about the mathematicians who had created the results that we were studying. After that semester was over, we became close friends, and this friendship, together with his comments, indicated that I had the talent to be a mathematician myself. This influenced me to reconsider my educational priorities, and it energized me to change my major from physics to mathematics. My strategy to successfully accomplish this transition was just like my high school experience in learning Calculus. Instead of focusing attention on my classes, I spent most of my days in the U. C. Berkeley math library reading the research journals, especially the *Annals of Mathematics*, which is considered to publish the most important works in the field. In that first six month period of journal study, I believe I learned about half of all the math that I presently know, although I must admit that I did not understand this newly acquired knowledge all that well!

I also looked for mentors and became acquainted with several of the best faculty in the Berkeley Math Department. I grew especially close to Steve Smale, who had recently won the Fields Medal for his work on the Poincare Conjecture in dimensions greater than 4. Steve took a special interest in me, and one day he invited me to drive along with him to a colloquium talk he was giving at Stanford University. I said “Yes” and we wound up discussing math non-stop for the entire trip, talking about what inspired him to find his proof of the h-Cobordism Theorem, about our common interests in dynamical systems, and about his work on the index of elliptic operators. As you might imagine, good fortune had settled on me by this time.

**Newsletter:** Tell us a little bit about your graduate studies, your initial appointment at UCLA, and the mathematics you did while there.

**Meeks:** Later in graduate school, also done at Berkeley, I became friends with Professor Mark Green who once asked me to teach his class on Hodge Theory during a two-week period while he was traveling. Although Mark was not my thesis advisor, he was the one who taught

me much of the material that influenced my thesis work with Professor Blaine Lawson. It was Mark, especially, who influenced my taste in research with his explanations of material in complex algebraic geometry. Just before I graduated from Berkeley in 1975, Mark helped me to get my first job as a Hedrick Assistant Professor at UCLA.

At the end of my second year there, I met S. T. Yau, who had just solved the important Calabi Conjecture in complex algebraic geometry. He told me that he and his five students would be attending my course on the topology of three-dimensional manifolds in the Fall Semester of 1976. After the course, Yau asked me if I thought one could use the course material to solve the following famous conjecture in geometry: if a simple closed curve in 3-space lies on the boundary of a convex set, then this closed curve is the boundary of an embedded (does not self-intersect) disk of (finite) least area. This problem has its roots in the famous Douglas-Rado solution to the classical Plateau Problem. A couple of weeks later, I realized that the topological methods in my course could probably be adapted to completely solve this very general problem, and eventually, in joint work with Yau, the conjecture was affirmed.

As typically happens with the best new results in mathematics, our solution to this problem of embedded least-area disks led to the solution of a seemingly unrelated classical problem – the Smith Conjecture – in a different mathematical field. Here is a brief idea of this conjecture. Suppose that  $R$  maps 3-space into itself via a rotation by angle  $\frac{k\pi}{n}$  around an oriented line  $L$  in 3-space, for some positive and relatively prime integers  $k$  and  $n$ . Then it is easy to check that  $R$  satisfies: (i)  $R$  is a 1-1 correspondence of 3-space with itself; (ii)  $R$  has finite order – in other words, there is a positive integer  $m$  such that the composition of  $R$  with itself  $m$  times is the identity map on 3-space; (iii) the mapping  $R$  and its inverse mapping (rotation by the angle  $-\frac{k\pi}{n}$  around  $L$ ) are both infinitely differentiable and preserve the orientation of 3-space (to preserve the orientation just means that the determinant of the derivative matrix is positive at every point in 3-space). A mapping satisfying these properties is called an orientation-preserving diffeomorphism, and the set of all such diffeomorphisms forms a group under composition. Our work on Plateau's problem led naturally to the final step in the proof of the generalized Smith Conjecture in topology, which states that every diffeomorphism of 3-space with finite order is obtained by conjugating (with some fixed diffeomorphism) a rotation  $R$  around a line.

**Newsletter:** With this meteoric beginning, tell us how your research continued.

**Meeks:** After this work with Yau, I became a world-traveler and lecturer. My travels took me to many beautiful places: Rio de Janeiro, Paris, Princeton and the San Francisco Bay Area, to do research with some of the best minds in my subject of differential geometry. Some of the key coauthors of my best mathematical papers were S. T. Yau (now at Harvard University), Harold Rosenberg (now retired from University of Paris VII, and visiting IMPA in Brazil), Charlie Frohman (now at the University of Iowa) and the late Nicholas Kuiper (former director of IHES, the research institute outside of Paris). Before and after coming to UMass, I wrote several key papers with my colleagues David Hoffman (now retired from UMass and visiting Stanford) and Rob Kusner. And more recently, I have been collaborating extensively with Antonio Ros and Joaquin Perez at the University of Granada.

**Newsletter:** In your future mathematical work, what is it that you see doing?

**Meeks:** I'm glad you asked that question. Let me tell you about my favorite problems in mathematics that I would like to work on – and hopefully solve! My lifetime goal has always been to discover the mathematical secrets of the Universe. This explains my early curiosity about theoretical physics and my interest in the close connection between differential geometry and modern physics: as you will recall, differential geometry played a major role in Einstein's general relativity theory. I have always tried to work on the important and difficult problems in my areas of interest, and this continues today. In the near future, the great geometry problems that attract me include the Hodge Conjecture in complex algebraic geometry, the Frobenius Conjecture in complex analysis, the complex Poincare-Bendixon Conjecture, the Embedded Calabi-Yau and Hoffman-Meeks conjectures about minimal surfaces, and last – but certainly not least – the complete classification of compact three-dimensional manifolds. Success on any of these problems would represent great progress for mathematics!

As I mentioned before, I began my journal study reading the *Annals of Mathematics*. Presently I have five published works in this journal, and a sixth paper (with Perez and Ros) was just accepted there. It has always been a personal goal to publish several more papers in this prestigious journal. That alone is a great incentive to keep working in a mathematical area of general interest!

By contrast to all that we've discussed so far, I also enjoy teaching vector calculus to our young students. Using computer-prepared materials, we can visualize some complicated and difficult-to-draw surfaces in 3-dimensional space, so the students can see more clearly the relationships between tangent planes, gradient vectors, curvature of surfaces, and

so forth. Of course, one can also see an immediate connection – on an elementary level – with my research interests.

**Newsletter:** Thanks, Bill, for this candid discussion of your work and – most especially – for your boundless enthusiasm about the field of geometric analysis!

Oblomkov article continued from cover

bridges to other mathematics departments in the valley by having two of the students come from Smith and Mount Holyoke colleges. The grant also provides summer support for a graduate student, who will work with Oblomkov and the undergraduates on the REU projects. According to Alexei, "I will work with the graduate student during the Spring to prepare him or her for mentoring the REU students. Then, after first month of the REU, my grad student and I will split the group of undergrads so that we can each work more closely with our own group."



As the title of Oblomkov's award suggests, his current research interests are in knot theory. Unlike many technical objects in mathematics, a mathematical knot is exactly what one imagines it to be – a loop in three dimensions: take your shoelace, make a loose knot with it, and join its ends with tape to form the loop; most likely you'll get something you can't wiggle around and turn into a lace without a knot on it, unless of course you cut the lace!

One of the main goals in knot theory is the exploration of knot invariants. A knot invariant is an algebraic or arithmetic or geometric quantity that helps to distinguish different types of knots from each other, or to tell when a knot is indeed knotted: suppose somebody gives you a terribly knotted shoelace, with the ends joined, and asks you to decide whether you can unknot it; one very effective knot invariant uses an algorithm that allows you to take pictures of "pieces" of the knot, put them in

the computer, and produce tables of integers – if any are nonzero, you cannot unknot the knot!

These integers, and their fancier cousins, reflect deep and subtle relationships between the geometry of knots and many other areas of mathematics: algebraic geometry, representation theory, mathematical physics – even number theory – all make an appearance.

Oblomkov's work sits at the intersection of all these topics and more. One reason for this is that Alexei has always been interested in the relationships between different mathematical areas: "What I like most in mathematics is the quest for the connections between the fields and how you often have to go to another field to find a key to your problem."

Unlike many mathematicians, who knew from an early age that mathematics was their passion, Alexei came to mathematics by accident. "Initially I wasn't interested in pure mathematics at all. I started as mechanics major at Moscow State University because of my fascination with physics, gas and fluid dynamics especially." But the demise of the Soviet Union meant that support for experimental science deteriorated, and he turned to theoretical physics. "Soon I discovered that the problems from mathematical physics lead to complicated systems of algebraic equations, and thus I decided to take a course in algebraic geometry with the hope of finding tools for my physics problems."

Today Alexei continues his brand of "applied" pure mathematics by emphasizing the computational aspects of his areas. "The practical and applied aspects of mathematics have great appeal to me. In particular, I use a lot of software in my research, and I always think of solving a problem as attempting to write some effective computational algorithm for that problem."

## OUTSTANDING UNDERGRADUATES HONORED AT 2014 AWARDS DINNER

On April 7th, 2014, the Department of Mathematics and Statistics celebrated the achievements of our top undergraduates at our annual Awards Dinner. This evening is held to recognize the winners of the Jacob-Cohen-Killam Mathematics Competition and the M.K. Bennett Geometry Award, as well as our REU participants, honors thesis students, members of the Putnam Competition team, and other students deserving special recognition. Along with the family and friends of the awardees, this year we were joined by alumni **John Baillieul '67**, **James Francis '86**, **Roy Perdue '73**, and faculty member emeritus **Eleanor Killam**.

The evening began with the customary refreshments and dinner; this year, each of the tables was decorated with an icosahedron, the most perfect Platonic solid. The awards portion of the event opened with greetings from department head **Michael Lavine**, who welcomed the audience to this year's Awards Dinner. The evening continued with remarks by this year's emcee, Professor **Jenia Tevelev**, who emphasized the unique history of mathematics, and described how knowledge has been passed down from teacher to student in an unbroken chain for over two thousand years.

**Morgan Opie** was recognized for winning a **Rising Researcher Award**, a university-wide honor celebrating undergraduates who demonstrate leadership and impact in their research. She was also acknowledged as the Runner-up for the **Alice T. Schafer Prize**, a national prize awarded to undergraduate women for excellence in mathematics. Finally, Opie was also honored for receiving a **Churchill Scholarship** for a year's graduate study in mathematics at Cambridge University.

**The M.K. Bennett Geometry Award** is presented to the student who exhibits the best performance in Math 461. This award honors the memory of Professor **Mary Katherine Bennett**. In 1966, Professor Bennett earned the first Ph.D. in our department. After teaching at Dartmouth College, she returned to UMass Amherst for the rest of her career where she encouraged interest in geometry and high school teaching among undergraduates. The year-long course she developed is now known as Math 461: Affine and Projective Geometry, and Math 462: Geometry II.

**Morgan Opie** was awarded this year's prize by Professor **Jenia Tevelev**, who presented Opie with two texts: Mumford's *Red Book*, the algebraic geometry primer, and Euclid's *Elements*, the most reprinted book in the world after the Bible.

**Aiden Hall**, **Andrew Mauer**, **Morgan Opie**, and **Andrey Smirnov** were recognized for competing in the **2014 Putnam Exam**, as were **Morgan Opie**, **Sarah Robertson**, **Thananya Saksuriyongse**, **Alexandra Saracino**, **Victoria Wang**, and **Ziwen Wang** who wrote **Honors Theses** this year.

This year's event particularly recognized the diversity and scope of our **REU projects**. Through the generous support of **Joan Barksdale '66**, this year the department was able to significantly expand our REU program. Three professors were present to discuss past REU projects, and describe how the REU program works in our department.

Professor **Farshid Hajir** introduced the REU students in Pure Mathematics, recognizing **Benny Chavez**, **Joshua Coutu**, **Ping Fung**, **Morgan Opie**, **Thananya Saksuriyongse**, and **Asherah Ravish**. The REU in Applied Mathematics were introduced by Professor **Matthew Dobson**, who recognized **Ian Fox**, **Kenneth Ottaviano**, **Alexandra Saracino**, and **Ziwen Wang**. Finally, the REU in Statistics were introduced by Professor **Krista Gile**, who recognized **Yan Ling**, **Sam Richardson**, and **Sarah Robertson**.

**The Jacob-Cohen-Killam Mathematics Competition** is named in honor of the memories of Professors **Henry Jacob** and **Haskell Cohen**, and of the continuing contributions of Professor Emeritus **Eleanor Killam**. These three faculty members encouraged interest in mathematics among undergraduates through annual mathematics contests.

The Competition is open to all first-year and second-year students who do not have a family association within the department. Each year, a few dozen contestants attempt to solve ten challenging problems posed by our faculty members. Former contestants tend to develop deep ties with the department; some choose mathematics as a double major, while others participate in summer REUs or join the Putnam team. This year **John Baillieul '67** joined long-time supporters **Roy Perdue '73** and **James Francis '86** in sponsoring the competition.

Professor **Rob Kusner** awarded this year's first prize of \$1600 to **Batkhuayag Batsaikhan**; the second place prize of \$800 each was shared by **Aaron Dunbrack** and **Da Lu**, while the 4th place prize of \$200 went to **Jeremiah Davis**.

The evening ended with closing remarks by **Steve Goodwin**, Dean of the College of National Sciences. The Dean spoke warmly of the successful evening, inviting **Morgan Opie** and Professor Emeritus **Eleanor Killam** to stand with him on the stage to highlight the valuable legacy and bright future of women in our department.



Roy Perdue



Rob Kusner, Da Lu, John Baillieu, Bathuyag Batsaikhan, Roy Perdue, Eleanor Killam, Jeremiah Davis and Aaron Dunbrack



James Francis



Matthew Dobson and Alexandra Saracino



Da Lu, John Baillieu, Bathuyag Batsaikhan



Andre Smirnov and Andrew Maurer



Ziwen Wang and Da Lu



Michael Lavine, Krista Gile, Ziwen Wang, Sarah Robertson, Alexandra Saracino, Ian Fox, Paul Hacking, Matthew Dobson, Farshid Hajir, Morgan Opie, Jenia Tevelev



Aiden Hall, Andrey Smirnov, Morgan Opie and Andrew Mauer

## GRADUATE PROGRAM NEWS

The Ph.D. program had a banner year, with twelve students receiving their degrees between September and May: **Nico Aiello, Dechang Chen, Holley Friedlander, Alden Gassert, Daniel Herr, Anna Kazanova, Jennifer Koonz, Jingran Li, Kai Li, Luke Mohr, Julie Rana, and Dong Yan.** That is more than we have ever had in one year, and we are delighted with all of their achievements. In addition, fourteen students received MS degrees: **Daniel Briggs** and **Dugan Hammock** in mathematics, **Matthew Brown** and **Christopher Rocheleau** in applied mathematics, and **Mei Duanmu, Derek Mobilio, Tanner Parente, Anthony Scotina, Ngoc Thai, Jian Tian, Dong Yan, Fei Yu, Nan Wang, and Zhe Zhao** in Statistics. Congratulations to all of these students!

In January, **Nico Aiello, Elizabeth Drellich, Alden Gassert** and **Julie Rana** attended the AMS-MAA Joint Mathematical Meetings in Baltimore. Elizabeth and Alden gave talks in special sessions there.

**Elizabeth Drellich** presented a poster at the Discrete Math Day at Wesleyan University and gave talks at AMS sectional meetings at Temple University and Washington University, as well as locally at UMass in the Valley Geometry Seminar and the GRASS graduate student seminar. She also posted a preprint based on part of her thesis.

**Kai Li, Haitao Xu, and Dong Yan** were each coauthors on two preprints with Professor Panos Kevrekidis and various other collaborators.

**Nico Aiello** gave a talk at the Maine-Quebec Number Theory Conference in Orono, Maine, which **Matthew Bates** and **Emma Dowling** also attended.

Many of our students attended one or both of the AGNES algebraic geometry conferences, held at Boston College and Stony Brook University this year, including **Matthew Bates, Nikolay Buskin, Zhijie Dong, Anna Kazanova, Huy Le, Julie Rana, Tassos Voggiannou, Bradley Willocks, and Feifei Xie.**

In January, **Jeff Hatley** attended the MSRI workshop “Hot Topics: Perfectoid Spaces and their Applications,” and in

March he went to the Arizona Winter School on Arithmetic Statistics.

This summer, **Matthew Bates** attended the Seminaire de Mathematiques Superieures 2014: “Counting Arithmetic Objects” in Montreal, and MS student **Domonic Mei** attended the MSRI summer graduate school in Dispersive Partial Differential Equations. Both of these were made possible through the department’s affiliation with MSRI.

In April, **Matthew Bates, Richard Buckman** and **Andrew Havens** attended the Graduate Student Topology & Geometry Conference at the University of Texas, Austin. In July, Richard attended the 2014 West Coast Algebraic Topology Summer School at the University of British Columbia.

**Kostis Gourgoulis** presented a poster at the Multiscale Computational Methods in Materials Modelling tutorial and workshop in Edinburgh.

This August, **Isabelle Beaudry** gave a talk, and **Weilong Hu** is presented a poster, at the Joint Statistical Meetings in Boston.

This summer **Toby Wilson** participated in The Data Incubator, a “boot camp” program which trains science PhDs for careers as data scientists and quants.



Isabelle Beaudry

### Distinguished Thesis and Teaching Awards

This year, in light of the unusually large number of students who graduated, the Graduate Affairs Committee chose three students to receive the departmental distinguished thesis award: **Alden Gassert, Luke Mohr** and **Julie Rana.**

**Alden Gassert** defended his thesis in April under the direction of Professor Farshid Hajir. Alden’s thesis, which is in the area of arithmetic dynamics, concerns the action of Chebyshev polynomials and related polynomials on finite fields. In his nominating letter, Professor Hajir stresses

Alden's growing strength and independence as a researcher, and says that by the end "Alden was now truly the expert and I could just sit back and watch the new ideas develop." He has written up the first two parts (of three) of his thesis; the first part was published this year in the journal *Discrete Mathematics*, and the second has been submitted to a journal. He has accepted a two-year postdoctoral position as a Burnett Mayer Instructor at the University of Colorado, Boulder. He will be a project NExT fellow for 2014-15.



Luke Mohr celebrating the completion of his PhD with Professor HongKun Zhang

**Luke Mohr** defended his thesis last August under the direction of Professor Hongkun Zhang. In his thesis, he solved a very important question related to understanding the superdiffusion phenomenon using the dynamical systems point of view. While previous work (of top people in the field) dealt with

processes generated by some very special dynamical systems, Luke's thesis gives a proof for the most general case. Professor Zhang praises the "elegant approach" he used and writes that "the results here will become classical theorems in mathematical physics in understanding superdiffusions." Luke continued at UMass as an instructor last year.

**Julie Rana** defended her thesis in December under the direction of Professor Jenia Tevelev. Her thesis studies the boundary of the Kollár-Shepherd-Barron-Alexeev moduli space of stable complex algebraic surfaces, accounting for how smooth surfaces degenerate to more complicated objects.



Jenia Tevelev and Julie Rana

She completely described two kinds of boundary behavior in the case of degree five surfaces, the smallest degree for which the boundary is not known. In his nomination letter, Professor Tevelev praises her broad approach, which uses "an impressive range of mathematics varying from classical algebraic geometry (geography of algebraic surfaces) to higher dimensional geometry (minimal model program) with a focus on several flavors of moduli theory and deformation theory." Julie has presented results from her thesis at numerous conferences and seminars. In Fall 2013 she began a two-year teaching position as the Mathematics Fellow at Marlboro college in Vermont.

The departmental distinguished teaching award was given to **Toby Wilson**. He was nominated by Cat Benincasa, who pointed to "how much he cares about his teaching technique, assignments, as well as the personal progress of students," and noted that "he has gone outside of the class with his desire to reach students with mathematics by running the Math Club, doing extra hours in the Calculus Tutoring Center and holding review sessions for his as well as other students." Last semester



Toby Wilson

he and Nico Aiello co-taught an after-school Geometry course to mathematically advanced 7th graders at Amherst Regional Middle School. They worked with the ARMS curriculum director to design a course that would cover 9th grade geometry over a year and a half. The course merged traditional classroom teaching with independent and group work. Toby is currently writing up his thesis, and plans to defend it in late 2014 or early 2015.

In addition, last year's winner of the departmental teaching award, **Nico Aiello**, won the University-wide Distinguished Teaching Assistant award this year. This award was given to only two TAs across the entire campus! We are delighted to have the University confirm what we already knew – Nico has been an extraordinarily dedicated, talented, and effective teacher. Nico graduated in May and is starting a job at Bloomberg in New York City. Although he is leaving the academic track, he says that he hopes to find time to teach some classes on the side.

## Five College Teaching Fellows

In the past few years, we have frequently received requests from Amherst, Mount Holyoke, and Smith colleges for one of our graduate students to teach a class, and we have been happy to send them some of our best teachers. **Nico Aiello**, **Toby Wilson**, **Stephen Oloo**, and **Elizabeth Drellich**, among others, have done this. This helps the colleges fill unexpected teaching shortfalls, and it's also been a great thing for our students: they get a chance to experience a liberal arts teaching environment, particularly helpful if they apply for jobs at small colleges, as many of our students do.

**Michael Lavine**, **Ilona Trousdale**, and **Tom Braden** realized it would be wise to formalize this relationship, so Michael had discussions with math department heads at the other colleges. The result is the **Five College Teaching Fellowship**;



Steve Oloo

it will support up to one student per semester to teach at Amherst, Hampshire, Mount Holyoke or Smith. Students will be selected for the fellowship based on their experience and excellence in teaching. By creating this fellowship, we hope to increase the visibility and prestige of these teaching exchanges, and ensure that they continue into the future. **Stephen Oloo** (Fall 2014, Amherst College) and **Isabelle Beaudry** (Spring 2015, Smith College) are the first two Five College Teaching Fellows.

## NEW FACES IN THE DEPARTMENT

**R. İnanç Baykur** joined the department as an Assistant Professor in September 2013. He works in low-dimensional geometry and topology, with a focus on exotic structures on 4-manifolds and cobordisms between them. His research involves constructions of contact structures on 3-manifolds, as well as smooth, symplectic, and complex structures on 4-manifolds; he also studies 4-manifolds via various singular fibrations, where Gauge theory and mapping class groups play vital roles. İnanç received



his Ph.D. in 2007 under the supervision of Ron Fintushel at Michigan State University. He then held postdoctoral positions at Columbia and Brandeis universities, and immediately before coming to UMass, he was a Research Professor at the Max Planck Institute for Mathematics in Bonn. İnanç has published 20 research papers, and is the author of a book on the traditional game of checkers. He has organized several workshops and conferences on low-dimensional topology, and is currently co-organizing the weekly Geometry & Topology Seminar here at UMass.

**Christine Ingraham** became Assistant to the Department Head and Associate Head last fall. While new to the department, she is familiar with the University, having previously worked in the department of Languages, Literatures and Cultures. She is currently part of the comedy improv troupe *Ridiculi* and occasionally plays guitar.

**Jacob Lagerstrom** succeeded long-time Assistant to Undergraduate & Graduate Programs and Graduate Admissions, Chris Richotte, in August 2013. Prior to that, he was a Coordinator for Residential Life. He graduated from UMass with degrees in Economics and in Social Thought & Political Economy.



Jake plays drums in *The 413*, which won best new band in The Valley Advocate the year before last.

**Oliver Waldman** joined the department this May as Assistant to the Applied Math Center and travel preparer. He practiced family law in Oregon, and has also served as a consultant to the Massachusetts Department of Revenue.

Although Opie initially intended to study physics at UMass Amherst, she had the opportunity to take advanced classes in both physics and mathematics here, which cemented her interest in abstract math. Her area of focus – algebraic geometry – is the type of math that involves tuning mathematical parameters to create and investigate intricate geometric structures – a mathematical version of playing with a multi-dimensional kaleidoscope.

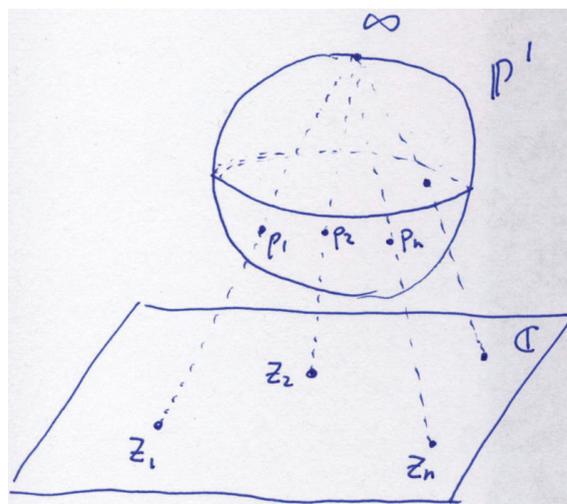
As a junior here, Opie took the year-long course in graduate-level algebra (Math 611-612) with Professor Tevelev. Impressed by Opie’s drive and curiosity in searching for novel solutions, Tevelev agreed to become her advisor and mentor during her remaining time at UMass Amherst. So, starting in March 2013, Opie prepared for the summer REU by reading Miles Reid’s *Undergraduate Algebraic Geometry* and Igor Shafarevich’s *Basic Algebraic Geometry*.

To paraphrase Claire Voisin, a distinguished French mathematician: algebraic geometry involves studying the variety of perspectives from which one can see the same object, using the “constant moving back and forth between several geometries and several types of tools to prove results in one field or another.”

One area of investigation for Tevelev has been the “moduli” space of stable rational curves with  $n$  punctures. This space is of special interest because it is the genus-zero case of the more-difficult-to-understand moduli space of algebraic curves of arbitrary genus, but its study has the potential to reveal information about the general moduli space. This is similar in spirit to how biologists might investigate simple worms in order to understand more complicated organisms, like humans.

Studying the moduli space of algebraic curves is an active and important area of research in algebraic and differential geometry, resulting in several Fields Medals – the most prestigious prize in all of mathematics – in recent years, including one in 2014 to Maryam Mirzakhani, the first female Fields Medalist ever!

The math story starts with  $n$  different complex numbers  $z_1, \dots, z_n$  – or, better still, add  $\infty$  to the complex line to obtain  $n$  distinct points on what is called the Riemann sphere in complex analysis, or the complex projective line  $P^1$  in algebraic geometry.



The Riemann sphere admits a rich group of conformal automorphisms, the Möbius transformations, given by fractional linear maps of the extended complex line having the form

$$z \rightarrow \frac{az + b}{cz + d}$$

where  $a, b, c$  and  $d$  are entries in an invertible complex 2-by-2 matrix (the value of this map is  $\infty$  when the denominator vanishes).

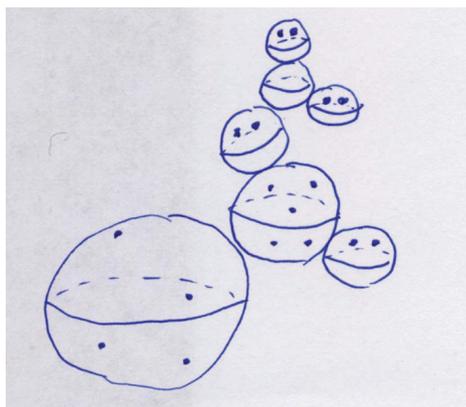
Two  $n$ -tuples of points are called *projectively equivalent* if there is a Möbius transformation that moves one to the other (analogous to the idea of triangles being *congruent* if there is a rigid motion taking one to the other). Now we can introduce the moduli space  $M_{0,n}$  which parameterizes  $n$ -tuples of points up to projective equivalence. For example,  $M_{0,3}$  is a point because any triple can be moved by a Möbius transformation to any other triple – in this geometry all triangles are equivalent!

But not so for all quadruples: they are distinguished by their cross-ratio

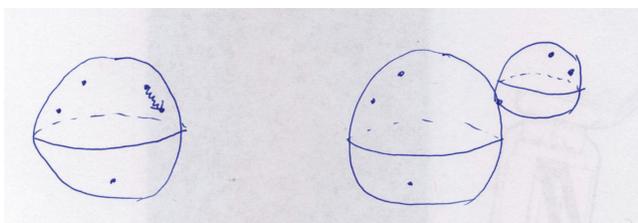
$$\frac{z_4 - z_1}{z_4 - z_3} : \frac{z_2 - z_1}{z_2 - z_3}$$

which is invariant under Möbius group. In fact,  $M_{0,4}$  can be identified with the set of all possible cross-ratios, which is any complex number except for 0 and 1.

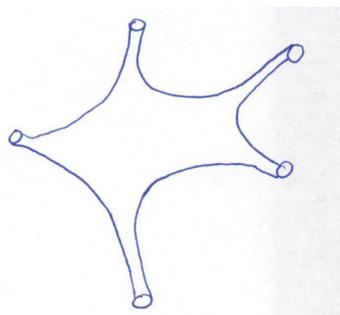
This simple example already shows that  $M_{0,n}$  is in general not compact. The reason for this is that points in  $M_{0,n}$  are not allowed to collide. This space has a natural compactification called  $\overline{M}_{0,n}$  or the *Deligne-Mumford compactification*. Just like a point of  $M_{0,n}$  can be visualized as a Riemann sphere with  $n$  punctures, a point of  $\overline{M}_{0,n}$  can be visualized as a tree of spheres with  $n$  punctures:



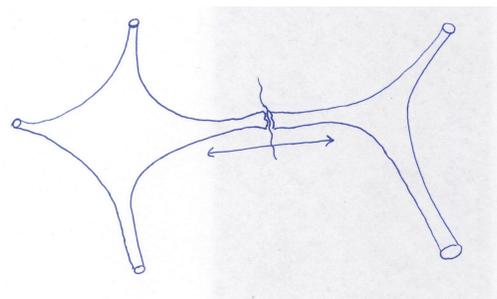
When 2 of the  $n$  points on the sphere try to collide, imagine watching their neighborhood under a microscope using stronger and stronger magnification to keep the points apart: the rest of the sphere moves further and further away. In the limit, this neighborhood becomes a new sphere carrying 2 distinct points, the original sphere carries the remaining  $n-2$  points, and these spheres are joined at a “node” where the 2 points had tried to collide on the original sphere:



By way of a very precise analogy, imagine an amoeba with five tentacles, in which two tentacles are trying to move far away from the other three tentacles:



In the limit, the amoeba fissions at the node into two amoebas:



When 3 or more points are involved, they can try to collide at very different rates, leading to the complicated trees of pointed spheres at the boundary of  $\overline{M}_{0,n}$ .

A big open question – and a source of some of Tevelev’s research – is how to describe all possible embeddings of  $\overline{M}_{0,n}$  into some high-dimensional projective space  $P^N$ . An even bigger open question is how to describe all possible hypersurfaces (also known as divisors) in  $\overline{M}_{0,n}$ . This question boils down to describing the boundary of a rather mysterious convex set (in a high-dimensional real vector space) called the effective cone of  $\overline{M}_{0,n}$ . A large set of points in this boundary was found by Castravet and Tevelev using hypertree divisors constructed in their Crelle paper “Hypertrees, Projections, and Moduli of Stable Rational Curves”, *J. Reine Angew. Math.* **675** (2013), 121-180. Opie had initially created a database of these hypertree divisors, but her main contribution has been the construction of new points on the boundary of the effective cone, inspired by an earlier paper “Extremal Effective Divisors on  $\overline{M}_{1,n}$ ” written by Dawei Chen and Izzet Coskun.

The most important parts of Opie’s REU project were already in place by July 2013, and her Capstone Senior Thesis was also based on her summer research, so Opie was ready to present a research poster at the Ohio State University Young Mathematicians Conference and the AGNES (Algebraic Geometry Northeastern Series) Conference at Boston University, as well as give seminars during the year at the University of Massachusetts and the University of Illinois at Chicago.

During her year of post-graduate study funded by the Churchill Fellowship, Opie will deepen her understanding of algebraic geometry and explore other areas of math, as well as “wrangle” for the Master of Advanced Study through

Part III of the Mathematical Tripos at the University of Cambridge. After the year in Cambridge, England, she plans to move to the Cambridge closer to home and pursue a PhD in Pure Mathematics at MIT.

As Tevelev remarks, “Opie’s drive and stamina became legendary in our department, where she is known as

a true force of nature.” These qualities, coupled with the research experience gained at UMass Amherst, will serve Mogan Opie well, both in graduate study, and beyond, in whatever mathematical adventures she pursues. We are pleased that Morgan chose to study mathematics with us, and we look forward to learning about her future accomplishments, which we expect will prove numerous.

## NEW CHALLENGE PROBLEMS

Our fall menu is full of new problems from the 2014 Jacob-Cohen-Killam prize exam! In recent years we prefixed the menu with an annual Awards Dinner hors d’oeuvre, but this holiday season we offer our hungry problem solvers a seven-course mental meal:

**Problem 1.** An Amherst farmer divides a square field ABCD by erecting a stone wall along the diagonal AC to form two triangular fields ABC and CDA. The farmer then decides to subdivide the triangular field ABC into two smaller fields of equal area using the shortest straight fence possible. Find – with proof – the length and location of this fence.

**Problem 2.** Let  $f(x)$  be a continuous real-valued function such that  $f(2014x) = f(x)$  for every real number  $x$ . Is it true that  $f(x)$  is a constant function?

**Problem 3.** In the equation  $[3(230 + t)]^2 = 492,a04$  find the integer  $t$  and the digit  $a$ .

**Problem 4.** The graph of  $y = x \sin \frac{\pi}{x}$  for  $0 < x \leq 1$  defines a bounded (but very wiggly) curve in the  $(x,y)$ -plane. Is the length of this curve finite or infinite?

**Problem 5.** Any  $2 \times 2$  matrix  $P = \begin{pmatrix} a & -b \\ b & a \end{pmatrix}$  is called *Pythagorean* if  $a$  and  $b$  are integers such that  $a^2 + b^2 = c^2$  for some integer  $c$ . Suppose  $Q = \begin{pmatrix} 235 & -411 \\ 411 & 235 \end{pmatrix}$ .

Show that  $P = Q^{2014}$  is Pythagorean.

**Problem 6.** An infinite chessboard has squares indexed by integer vectors  $\begin{pmatrix} m \\ n \end{pmatrix}$  in the plane. This problem has three parts:

a) A *Knight* moves by any one of these eight vectors:  $\left\{ \pm \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \pm \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \pm \begin{pmatrix} -1 \\ 2 \end{pmatrix}, \pm \begin{pmatrix} -2 \\ 1 \end{pmatrix} \right\}$ .

Show that a Knight can visit any square on this chessboard (that is, any integer vector in the plane is an integer linear combination of these eight vectors).

b) An “impaired” Knight – or *Knite* for short – can only make the first four moves:  $\left\{ \pm \begin{pmatrix} 2 \\ 1 \end{pmatrix}, \pm \begin{pmatrix} 1 \\ 2 \end{pmatrix} \right\}$ .

What fraction of the squares can a Knite visit?

c) Suppose a new chess piece – the *Mathprof* – makes these six (rather weird) moves:  $\left\{ \pm \begin{pmatrix} 2 \\ 0 \end{pmatrix}, \pm \begin{pmatrix} 0 \\ 3 \end{pmatrix}, \pm \begin{pmatrix} 5 \\ 5 \end{pmatrix} \right\}$ .

What fraction of the squares can a Mathprof visit?

**Problem 7.** Call a function *lucky* if it can be expressed as a sum of a polynomial function  $p(t)$  and a continuous periodic function  $q(t)$  with period  $2\pi$  (this means that  $q(t+2\pi) = q(t)$  for every  $t$ ). Show that any antiderivative of a lucky function is lucky.

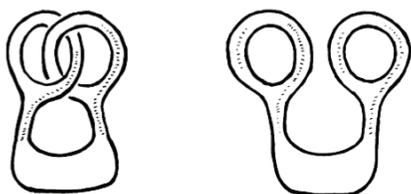
**We always like hearing from you.** Please send any solutions, comments, or other feedback via email <profkusner@gmail.com> with the subject line “Challenge Problems 2014” or via snail mail: Challenge Problems 2014, c/o Professor Rob Kusner, Department of Mathematics & Statistics, University of Massachusetts, Amherst, MA 01003. Remember to include your full name, information about your UMass Amherst degree(s) earned, and any other interesting things you wish to share with us.

## SOLUTIONS TO LAST YEAR'S CHALLENGE PROBLEMS

Your procrastinating Problem Master (PM) **Rob Kusner** thanks our prescient pair of problem solvers, **Karl David** (PhD '78) and **Mark Leeper** (BS '72), who were the only readers this year that shared their solutions – and they did that early last fall! Mark went further (sidebar) by sharing his Newsletter habits and his newsworthy post-1972 exploits.

Recall that we began with a topological challenge from the 2013 Awards Dinner program (essentially Problem 6 from last year's Jacob-Cohen-Killam prize exam) which involved a shape resembling a (very flexible) pair of spectacles:

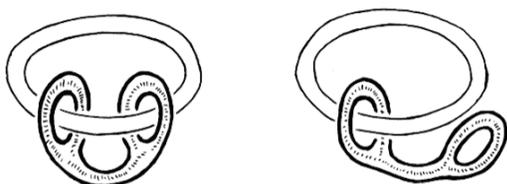
**Problem A.** Many puzzles sold in toy stores are based on topology, a branch of geometry where geometric shapes are allowed to “deform” (stretch, shrink, and twist, but not tear). Deformations can be highly non-trivial. For example, it seems very unlikely that the shape on the left (made of elastic rubber) can be deformed to the shape on the right:



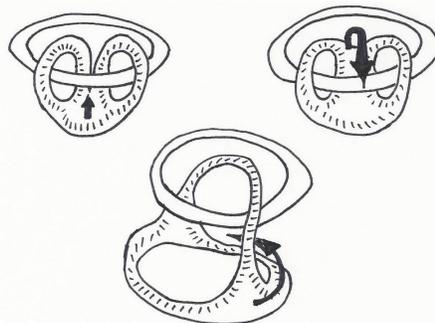
However, here is a sequence of deformation steps that does just that:



Problem A is to find and draw a sequence of deformation steps necessary to achieve the following seemingly impossible deformation:



Here's one way to do this which retains the left-right symmetry until the very last step:



This topological trick actually has an algebraic explanation using a change of generators for the “fundamental group” (faithful readers may remember that notion came up a few years ago when your PM and Professor **Eduardo Cattani** each wrote articles about the Geometrization and Poincaré Conjectures for 3-dimensional manifolds; though pursuing that would again go way beyond the scope of this Newsletter, interested readers are welcome to contact us for more details). ■

Now that we've untangled that one, let's sketch solutions to the other challenge problems, also adapted from the Jacob-Cohen-Killam competition. Mark may recognize our solution to the “zeroth” problem:

**Problem 0+.** Find  $\lim_{x \rightarrow 0^+} x^{(x^x)}$ .

Since  $x^{f(x)} = e^{f(x)\ln x}$  and  $\lim_{x \rightarrow 0^+} \ln x = -\infty$ , we have  $\lim_{x \rightarrow 0^+} x^{f(x)} = 0$  for any function with  $\lim_{x \rightarrow 0^+} f(x)$  positive. In particular, when  $f(x) = x^x = e^{x \ln x}$ , we have  $\lim_{x \rightarrow 0^+} f(x) = 1$ , because the L'Hospital Rule gives

$$\lim_{x \rightarrow 0^+} x \ln x = \lim_{x \rightarrow 0^+} (\ln x)' / (1/x)' = \lim_{x \rightarrow 0^+} (1/x) / (-1/x^2) = 0,$$

and therefore  $\lim_{x \rightarrow 0^+} x^{(x^x)} = 0$ . ■

Both Karl and Mark (independently, not collectively) solved the next algebraic and number theoretic one. In fact, Karl pointed out that his proof had “an intriguing serpent-biting-its-tail quality to it that instantly makes it one of [his] favorite shorter problems in recent years!” We streamline their solutions:

**Problem 1.** Let  $P(x) = x^2 + 2px + 2q$ , where  $p$  and  $q$  are odd integers. Show that  $P$  has no integer roots, and in fact no rational roots.

We have  $P(x) = x^2 + (4a + 2)x + (4b + 2)$ , provided  $p = 2a + 1$  and  $q = 2b + 1$  are odd, with discriminant

$(4a + 2)^2 - 4(4b + 2) = 4(4(a^2 + a - b) - 1)$ . This would be a perfect square for  $P$  to have rational (or integer) roots, and since 4 is perfect square, so would the other factor  $(4(a^2 + a - b) - 1)$ ; but it is not: even perfect squares are divisible by 4, while odd perfect squares leave remainder 1, not -1. ■

This year's election results are still rolling in as your PM turns to the next problem, for which the turnout was very low (in fact, none of our readers submitted solutions):

**Problem 2.** Utopia (population 10,236,688) is a Central American republic whose legislative authority is vested in a Senate made up of 100 senators elected from 100 senatorial districts. Voting is mandatory in Utopia. The Constitution of Utopia stipulates that a census be taken every ten years. After each census, the Senate can draw a new map of senatorial districts to ensure that each district has the same number of voters, but otherwise there are no restrictions. Utopia has two political parties, Applieds (As) and Pures (Ps). The first question: If As and Ps enjoy the same popular support, what is the maximal possible number of senators Ps can get by redrawing the senatorial map if they have a Senate majority after the census? Now suppose As have a Senate majority after the census but they are worried about losing popular support. The second question: If the As can keep tinkering with the senatorial map, what is the minimal percentage of voters they need to attract to stay in power forever?

To answer the first: observe that if the Ps have merely 1 extra supporter in each of 99 districts, then they will win exactly 99 Senate seats; the remaining seat will go to the As by a 99-vote margin. For the second: a majority means 51 Senate seats, and so if the As can tinker with – or *Gerrymander*, as it's known here in Massachusetts – the map to have over half support in those 51 districts, while packing Ps into the remaining 49 districts, just over 25.5% of overall voter support is all they need to hold the Senate forever. ■

These results may be relevant to other republics with similar electoral systems.

We hinted that the third problem (or was it the fourth?) might be more analytic than number theoretic:

**Problem 3.** Show that there is a perfect square of the form '77777...' with exactly 2013 '7's in the beginning.

Challenge Problem Solutions continue on page 25

## OUR FAITHFUL EXPERT PROBLEM SOLVER



Mark and Evelyn at the Cape of Good Hope, South Africa

I don't have a smartphone so I prefer the paper copy of the Newsletter because I can carry it around and work on problems at odd moments. [The margins on a smart phone wouldn't be wide enough anyway!]

### *What have I been doing since 1972?*

Okay, you asked for it! Well, I never succeeded in finding someone who would pay me to do math. I got a Masters in Mathematics from Stanford in 1974, worked for Burroughs for three years and then went with Bell Laboratories. That took me through AT&T, Lucent, and Avaya, which were all like working for the same company. In 2001, I retired young and am living (comfortably) on my investments. I devote myself to my hobbies. For a while I worked with Israel Gelfand at Rutgers, who took me under his wing, but I couldn't keep up with the math he wanted me to do.

At Bell Labs, my wife – Evelyn Chimelis '72, now Evelyn Leeper – and I founded and ran the science-fiction club, and even though it's not connected to the Labs any more, it still exists. We publish a weekly newsletter of about 6 to 8 pages each. That takes a lot of time. I'm the second-longest-running film reviewer on the Internet, reviewing films since the mid-1980s. I run a drop-in center at our town library for kids who want free mathematics tutoring. I do that for four hours a week. And I do a lot of fooling around with mathematics in my spare time. I probably should have gone further in mathematics, but I always wanted to do my own math rather than learning someone else's.

I have very fond memories of working with Dr. Samuel Holland, my best college professor. I hope he is doing well.

-Mark Leeper '72

REACHING DOWN THE RABBIT HOLE  
BY BRIAN BURRELL AND ALLAN ROPPER, M.D.

A new book, *Reaching Down the Rabbit Hole* by **Brian Burrell** and Dr. Allan Ropper, is being published this fall by St. Martin's Press. Brian is a senior lecturer in our department; Allan is Professor of Neurology at Harvard Medical School and the Executive Vice Chair of the Department of Neurology at Brigham & Women's Hospital in Boston. The Newsletter interviewed them this summer about their forthcoming book. Here's an excerpt from that interview:

**Newsletter:** How is it that a statistician, not directly involved in medical studies, and a practicing neurologist involved in treating severe mental disorders, wrote a book together?

**Brian:** My previous book, *Postcards from the Brain Museum: The Improbable Search for Meaning in the Matter of Famous Minds*, was being read by the members of the Neurology Book Club at Harvard, led by Allan. To my surprise, he called and invited me down to give a talk on my book [Brian's fifth published work]. Then Allan suggested we do a book together!

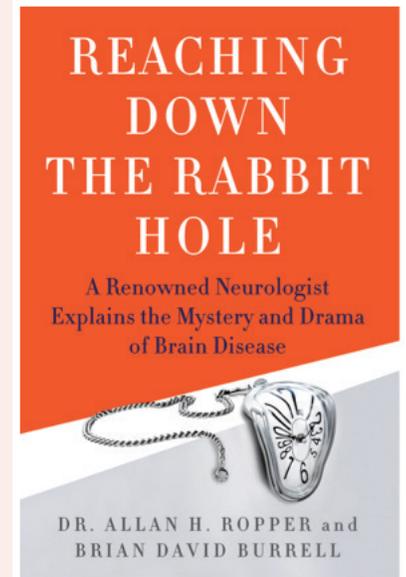
**Newsletter:** Allan, why did you want to write a book with Brian?

**Allan:** Just look at the diversity of topics in Brian's body of work and at his literary skill. But most important, he is a very interesting fellow. This is the kind of person you write a book with!

**Newsletter:** But how did you choose the topic?

**Allan:** After some discussion back and forth we suggested that Brian join the group of students and physicians in my hospital rounds.

**Brian:** Seeing firsthand how physicians treat severe and complicated mental illness is something people might like to read and learn about, especially from the physician's point of view. Thus, the book we have before us!



To give readers an idea of its contents, the Newsletter quotes from the St. Martin's Press pre-publication description of the book:

"Tell the doctor where it hurts." It sounds simple enough, unless the problem affects the very organ that produces awareness and generates speech. What is it like to try to heal the body when the mind is under attack? In this book, Dr. Allan Ropper and Brian Burrell take the reader behind the scenes at Harvard Medical School's neurology unit to show how a seasoned diagnostician faces down bizarre, life-altering afflictions. Like Alice in Wonderland, Dr. Ropper inhabits a world where absurdities abound:

- A figure skater whose body has become a ticking-time bomb.
- A salesman who drives around and around a traffic rotary, unable to get off.
- A college quarterback who can't stop calling the same play.
- A child molester who, after falling on the ice, is left with a brain that is very much dead inside a body that is very much alive.
- A mother of two young girls, diagnosed with ALS, who has to decide whether a life locked inside her own head is worth living.

How does one begin to treat such cases, to counsel people whose lives may be changed forever?

How does one train the next generation of clinicians to deal with the moral and medical aspects of brain disease?



Brian Burrell and Allan Ropper

Brian teaches our introductory statistics course, Stat 111. He is also working on his Ph.D. dissertation in Statistics with Professor **John Staudenmayer**. The Newsletter thanks both Allan and Brian for taking the time to discuss their new book, a chapter of which was excerpted in the October 12 *Boston Globe Magazine*.

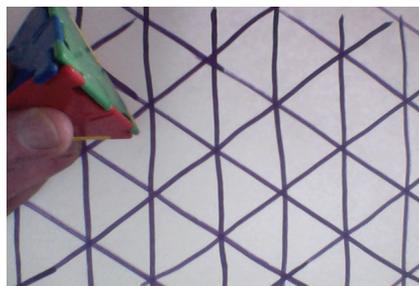
[www.bostonglobe.com/magazine/2014/10/12](http://www.bostonglobe.com/magazine/2014/10/12)

Challenge Problem Solutions continued from page 23

Mark offered a careful proof spiritually similar to – it was, in effect, the square-root of – the one your PM had in mind. The crucial observation is that the gaps between perfect squares grow arithmetically long, while the gaps between integers of the form  $77\dots7100\dots0$  and  $77\dots7200\dots0$  grow exponentially long (as a function of the number of trailing zeros). Thus there will eventually be (exponentially many) perfect squares of the desired form. ■

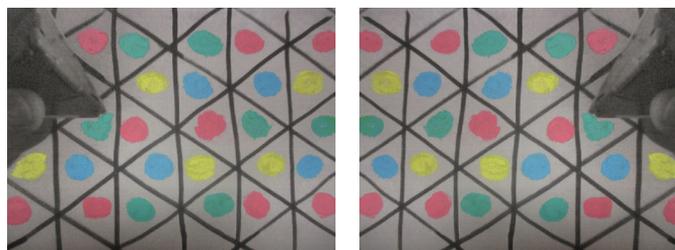
And we rounded out last year’s problems by rolling something that isn’t round:

**Problem 4.** A printer has a printing block in the shape of a regular tetrahedron, each of whose faces holds a particular color ink: Red, Green, Blue, Yellow. As this block is “rolled” on a big sheet of paper, it prints a periodic color pattern of equilateral triangles in the plane. Indicate with R, G, B, or Y or (or use crayons) a possible color pattern that one such block might print in the triangles:



Two patterns are distinct if one cannot be mapped to the other by scaling, translation or rotation of the plane. How many distinct patterns can arise from this printing process?

Consider permutations of the 4 colors on the block: half of these arise from rotations of the tetrahedron; the other half, from a rotation composed with a reflection. The first half will roll out the pattern (up to rotation and translation of the plane) depicted in the next figure; the other half, its mirror image. Thus there are two distinct color patterns:



■

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