

# The Graduate Programs in Mathematics and Statistics at the University of Massachusetts Amherst

Revised September 2021

## **Administrative Structure**

The Graduate Affairs Committee (GAC) has the ultimate authority over the Graduate Programs in Mathematics in Statistics. Although all graduate faculty may potentially be involved in the development and evaluation of a student, the GAC remains the supervisory body in all cases. The GAC is chaired by the Graduate Program Director (GPD). In practice, most routine matters are delegated to the discretion of the GPD. The GPD is assisted by the Graduate Program Manager (GPM). The GPD also serves as the academic advisor for all Mathematics PhD students prior to choosing a dissertation advisor. Applied Mathematics MS students are advised by the Applied Mathematics Director (AMD). Statistics MS and PhD students (prior to choosing a dissertation advisor) at the Amherst campus are advised by the Statistics Coordinator (SCA). Statistics MS students at the Newton campus are advised by the Statistics Coordinator–Newton (SCN).

Typically administrative matters should be taken initially to the GPM, who will route them appropriately. Academic matters should be taken initially to the student's academic advisor. More sensitive matters can be taken directly to the GPD if necessary. When in doubt, there is no harm in contacting the GPD to determine the best course of action.

## **Degrees and Requirements**

It is the student's responsibility to be familiar with the requirements and policies of the University and the Department, as stated in the Graduate School Bulletin and this handbook respectively. If a student is in doubt

about the application of these policies to their own case, the matter should be brought to the attention of the student's academic advisor.

The student should be aware of the general structure of the program as explained below, including the required coursework and the system of qualifying examinations. It is the student's responsibility to initiate each step in their degree program. Students must also be aware of the statute of limitations, as explained below and in the Graduate School Bulletin.

### **Degree Options**

The Department of Mathematics and Statistics offers PhD programs in Mathematics and Statistics and MS programs in Applied Mathematics and Statistics. It also grants an MS degree in Mathematics but this is usually only awarded as a step towards the PhD degree. Note that neither of the MS programs are primarily meant as preparation for a PhD program; a student who wishes to do a PhD after an MS in Applied Mathematics or Statistics must apply and be accepted to the PhD program like any other student. There is no formal distinction between pure and applied mathematics within the Mathematics PhD program.

A student who wishes to transfer from one degree option to another should notify the GPD as early as possible. Such a transfer requires the approval of the Graduate Admissions Committee, just as in the case of a new applicant, and will usually involve adjustments in the amount and/or duration of financial support as well as, in some cases, a formal application to the new program.

### **Degree Requirements**

Formal degree requirements are given below. Coursework requirements may be waived by the GPD if in their judgment the student has taken acceptable equivalents elsewhere. In some cases, strong performance on one of the written exams might be accepted as a substitute for taking a course. In other cases it may be possible to substitute an alternative course for a required course. To request a waiver of a course requirement, students should contact the GPD directly, explaining the course to be waived, the alternative course to be taken, if any, and the justification for the request. The GPD will return a decision granting or denying the waiver, and a copy will be placed in the student's file.

Note that while three credit independent study courses may be used to satisfy the minimum hours of coursework below, one credit seminars may

not be used for this purpose.

Students are reminded that in addition to Department requirements they must complete all degree requirements established by the Graduate School. Consult the Graduate School Handbook, available on the web page of the Graduate School. Degree application forms for an MS or PhD can be found on the Graduate School website as well. The student must fill out the form and return it to the GPM for verification and signatures of the GPD and the Department Head. The GPM will then submit the form and a cover letter to the Office of Degree Requirements.

### **MS Requirements**

Please note that the MS in Mathematics is usually only obtained by students within the Mathematics PhD program; there is no separate program awarding it.

#### **MS in Mathematics**

1. The student must complete 30 hours of coursework with grades of C or better, including at least 24 hours with grades of B or better. In addition, the student must have at least an overall B average.
2. Any course outside the Department or numbered less than 600 must have the GPD's approval to be counted toward the 30 hours. No more than four courses below the 600 level may be counted, and no more than 3 hours per course may be counted toward the 30 hours.
3. The required 30 hours must include 21 hours of Mathematics and Statistics courses (at least 18 hours numbered above 600), normally including at least four of the courses: Math 605, 611, 621, 623, 645, 651, 671, Stat 607.

#### **MS in Applied Mathematics**

1. The student must complete 30 hours of coursework with grades of C or better, including at least 24 hours with grades of B or better. In addition, the student must have at least an overall B average. (Note: Pass or fail courses cannot be used to satisfy this requirement.)
2. Any course outside the Department or numbered less than 600 must have the AMD's approval to be counted toward the 30 hours. No more

than four courses below the 600 level may be counted, and no more than 3 hours per course may be counted toward the 30 hours.

3. The required 30 hours must include at least 18 hours of Mathematics and Statistics courses, and must include:
  - (a) Math 532 (or 532H): Nonlinear Dynamics;
  - (b) Math 534 (or 534H): Introduction to Partial Differential Equations;
  - (c) Math 651: Numerical Analysis I;
  - (d) Stat 607: Mathematical Statistics I.

Each of the four courses can be replaced by an elective course if a student has taken an equivalent course before entering the program and with the approval of the AMD.

4. The student must complete a project in Applied Mathematics under the guidance of a faculty member. This project must have prior approval of the AMD and normally involves 3 credit hours (the credits earned may be used to satisfy the 18 hour requirement in (3)). The project might involve reading some research papers, analyzing some real data, and doing some computer programming.

### **MS in Mathematics (Statistics Option)**

1. The student must complete 30 hours of coursework, with grades of C or better, including at least 24 hours with grades of B or better. In addition, the student must have at least an overall B average.
2. Any course outside the Department or numbered less than 600 must have the approval of the student's Statistics Coordinator to be counted toward the 30 hours. No more than 3 hours per course may be counted toward the 30 hours.
3. The required 30 hours must include:
  - (a) Stat 625 (Regression Analysis);
  - (b) Stat 607–608 (Mathematical Statistics I, II);
  - (c) Stat 535 (Statistical Computing);

- (d) At least five other courses which are either Statistics courses numbered 526 or above, from within the department, or some courses outside the department numbered 500 and above subject to prior approval by the student's Statistics Coordinator.

Students are expected to have a background in advanced calculus and in linear algebra. Students with weak background in these areas will be required to take appropriate courses at an early stage of their graduate study here.

- 4. The student must complete a project in statistics under the guidance of a faculty member. This project must have prior approval of the student's Statistics Coordinator and involves 3 credit hours which may be used to satisfy the 30 hour coursework requirement. The project can take many forms; an expository report on a particular area, an examination of methods through simulations or a detailed statistical analysis of real data. A final report is required. This requirement is typically satisfied by the successful completion of the project seminar course Stat 691P.
- 5. The student must pass two of the three Basic exams in Applied Statistics, Probability and Statistics.

## **PhD Requirements**

### **PhD in Mathematics**

- 1. The student must complete successfully 36 hours of coursework, including:
  - (a) At least three full year sequences (Math 605–606, 611–612, 623–624, 645–646, 671–672, Stat 607–608);
  - (b) Math 621;
  - (c) At least one of Math 611, Math 623, Math 671;
  - (d) At least one of Math 645, Math 646, Math 651;
  - (e) At least one of Math 605, Math 606, Stat 607.
- 2. The student must pass the Advanced Calculus/Linear Algebra exam.
- 3. The student must pass two written exams chosen among Algebra, Analysis, Applied Mathematics, Stochastics and Topology.

4. The student must form an oral exam committee and pass the oral exam as described below.
5. The student must write a satisfactory dissertation and pass a final oral examination (primarily a defense of the dissertation), and must satisfy all other requirements of their dissertation committee. The student is required to register for a minimum of 18 dissertation credits.

### **PhD in Mathematics (Statistics Option)**

1. The student must complete successfully 36 hours of coursework, including Math 523 (or Math 623, or Math 605), Stat 535, Stat 607, Stat 608, Stat 625, Stat 705, and Stat 725.
2. The student must also complete five elective courses, including two 600 level statistics courses, and 3 courses of the student's choice, which require prior approval by the statistics coordinator.
3. The student must pass three Basic Exams at the PhD level in Applied Statistics, Probability and Statistics. (These cover the material from Stat 535 and Stat 625, Stat 607 and Stat 608 respectively.)
4. The student must pass the advanced statistics exam and the oral literature-based exam. These exams are described below.
5. The student must write a satisfactory dissertation and pass a final oral examination (primarily a defense of the dissertation), and must satisfy all other requirements of their dissertation committee. The student is required to register for a minimum of 18 dissertation credits.

### **Dissertation Committee, Prospectus, Dissertation Credits**

This section applies to all PhD students.

Upon completion of all course and exam requirements for their program, the graduate student should select a dissertation advisor from the faculty in the Department as soon as practical. Most often the advisor will have been the chair of the oral exam committee, but this is not required. Upon selecting a dissertation advisor and confirming this choice with the advisor, the student should notify the GPM, who will in turn send a confirmation notice to the student, advisor, GPD and Director of Administration and Staff.

The PhD candidate must form a Dissertation Committee, consisting of a dissertation advisor together with three other graduate faculty members (one of whom must be from another department on campus). The Department requires that the chair of the dissertation committee be graduate faculty in the Department of Mathematics and Statistics. Students are encouraged to form their dissertation committee as soon as possible after selecting their dissertation advisor.

The committee should advise the student about any expected need (in dissertation research or later) for reading material in a foreign language such as French or German.

A written prospectus of the proposed dissertation research must be drawn up with the advisor's input and signed by all committee members. This prospectus must be submitted to the Graduate School and the dissertation defense must be scheduled at least one month before the actual date of the dissertation defense. A L<sup>A</sup>T<sub>E</sub>X form for this purpose may be obtained from the GPM.

After passing all exams and choosing an advisor the student may enroll for dissertation credits: a minimum of 18 such credits are required for graduation.

### **Statute of Limitations**

This is the maximum time allowed for completion of a degree. A PhD student has six calendar years from acceptance into the graduate program, and a MS student four calendar years, to earn the degree. In exceptional circumstances, these limits may be extended with the approval of the GPD and the Dean of the Graduate School. Please note that these time limits are completely independent of any funding decisions.

All PhD students must

- Choose an official dissertation advisor (and notify the GPM about the choice) within one year after the completion of all exam requirements, or the start of the sixth semester, whichever comes last.
- Form a dissertation committee and submit a dissertation proposal (prospectus) to the Graduate School within 18 months after the completion of all exam requirements, or the start of the seventh semester, whichever comes last.
- If a student fails to satisfy either one of these conditions, at the discretion of the GPD the student could be granted an extra semester to satisfy these requirements.

- If a student fails to satisfy these requirements within the prescribed time period, the GAC in conjunction with the Department Head will review the student's status and future financial support.

## Coursework

Normally, a MS student or a PhD student who has not yet passed all exams takes 3 courses (9 credits) per semester. Taking fewer courses can be justified only under special circumstances, and requires the prior approval of the student's academic advisor as well as the GPD. Occasionally a well-prepared student may elect to take four courses, but this should be attempted only after consulting with the academic advisor. Audited courses do not count towards the 9 credit limit.

PhD students who have passed all exams and declared a dissertation advisor may fill out their schedule by signing up for dissertation credits. In some circumstances students at this level opt instead to request overrides to full time status from the GPD. Although the department allows this in some cases, it takes no responsibility for any financial complications that may result.

Most graduate courses are numbered 600 and above: students are generally expected to take at least two courses at this level each semester in the Department. Students are encouraged to take courses in other departments, provided the level and content are appropriate: permission to do this must be obtained in advance from the GPD if the student intends to use such courses to fulfill any degree requirements.

There are a number of courses at the 500 level, which are open to both undergraduates and graduate students. Some—but not all—of these courses are normally permitted to be used as part of the graduate program (e.g., Math 523, 532, 534, 545, 563), following GAC guidelines. Such courses are typically at a lower level than the introductory graduate courses, and are useful primarily for students seeking a MS. (Please note that the degree requirements above limit the number of 500 level courses which can be used.) Statistics courses numbered 525 and below are never permitted to be used for requirements by our graduate students.

With some exceptions, 600-level courses are offered every year. Most are part of two semester sequences. There are several 700-level courses which are offered every year or every other year. In addition, every year the Department will offer some topics courses which may not recur in the near future. (Please note that some of the course numbers below have not yet

been approved by the registrar and thus the course may be running with a temporary number for the time being.) The most accurate resource for course offerings in a semester is the Department web page or SPIRE.

### **600-level courses**

- Math 605/606: Probability / Stochastic Analysis
- Math 611/612: Algebra I / Algebra II
- Math 621: Complex Analysis
- Math 623/624: Analysis I / Analysis II
- Math 645/646: ODEs, Dynamical Systems / Applied Mathematics
- Math 651/652: Numerical Analysis I / Numerical Analysis II
- Math 671/672: Topology / Algebraic Topology
- Stat 607/608: Mathematical Statistics I / Mathematical Statistics II
- Stat 610: Bayesian Statistics
- Stat 625: Regression Modeling

### **700-level courses**

- Math 703: Differential Geometry
- Math 704: Riemannian Geometry
- Math 705: Symplectic Topology
- Math 706: Stochastic Calculus
- Math 707: Algebraic Geometry
- Math 708: Complex Algebraic Geometry
- Math 713: Algebraic Number Theory
- Math 714: Arithmetic of Elliptic Curves
- Math 717: Representation Theory
- Math 718: Lie Algebras

- Math 725: Functional Analysis
- Math 731: Partial Differential Equations
- Stat 705: Linear models
- Stat 725: Estimation Theory and Hypothesis Testing I

### **Independent Study**

Students may be permitted to pursue independent study with the guidance of a faculty member (in place of 3 or 6 hours of regular courses). The student should obtain the Independent Study form from the GPM, to be approved by the supervising faculty member and the GPD.

### **Dissertation credits**

Students in the PhD program must take a total of 18 credits of dissertation before they graduate. Dissertation credits can be taken at any time after the student has passed all exams and chosen a dissertation Adviser. Any number of dissertation credits can be taken in a semester, up to a maximum of 9 credits of dissertation and regular courses. For example, a student who is taking one 3 credit course can sign up for up to 6 credits of dissertation.

### **Grades**

The only grades which graduate students may earn are A, A-, B+, B, B-, C+, C, F. Grades below B are generally regarded as indicating substandard performance, as reflected in our degree requirements. According to Graduate School policy, a student whose average falls below 2.8 (where A=4, A- = 3.7, B+ = 3.3, B=3, C=2) in any two semesters (consecutive or not), is subject to academic dismissal upon recommendation of the GPD to the Dean of the Graduate School. But in more down-to-earth terms, a student who is unable to maintain a B average in the first two years of graduate work cannot expect to earn even a MS in our Department. Only in exceptional circumstances will a student whose average falls below B in any semester receive financial support.

### **Language Requirement**

English is currently the international language of mathematics and statistics, so all graduate students are expected to have a good mastery of English,

both written and spoken. Students whose first language is not English must show adequate reading comprehension as measured by the TOEFL exam. But fluency in the spoken language is also essential, for participation in courses and seminars or for employment as a Teaching Assistant. Students who have difficulty with the spoken language are expected to enroll in a conversation course or the equivalent. Although the department has no formal foreign language requirement, students are encouraged to acquire, at least, a reading knowledge of other languages widely used in mathematics and statistics.

### **Computer Literacy Requirement**

The department has no formal computer literacy requirement. However, students are encouraged to take advantage of the computing resources provided for numeric and symbolic computation, statistical data analysis, text processing, etc.

### **Qualifying Exams**

Each of the four graduate programs within the Department of Mathematics and Statistics has its own distinct examination requirements. Written qualifying examinations are offered in the week before the beginning of the fall and spring semesters. Sign ups are offered during the previous semester. Students new to the program may sign up for examinations by emailing the GPM, although we can not guarantee that the exam schedule will accommodate their plans. In the fall semester, the Advanced Calculus/Linear Algebra exam is offered during the second week of classes instead of the week before.

The examination committee for each exam will report the results in detail to the GAC, which will then make the final decision about passing or failure. In some cases when a student does not pass, but in the opinion of the GAC, no purpose would be served by requiring the student to retake all or part of the exam, the GAC may offer alternative ways for the student to qualify for the PhD. These could include: an oral exam (which may cover additional material), additional coursework, or potentially other options as well.

A student who changes programs within the Department may apply past exam results to the new program. For example, a student who moves from the Statistics MS program to the Statistics PhD program may apply any

Basic exams which they passed at the PhD level to their PhD program exams.

Syllabi for the exams detailed below are contained in the Graduate Handbook. Past exams may be found on the department web page.

Failure to complete any exams by the time limits given below will result in termination of the student in their program at the end of that semester (usually the spring semester).

### **Applied Mathematics MS**

The Applied Mathematics MS program currently has no examination requirements.

### **Statistics MS**

Students in the Statistics MS program must pass two exams at the MS level from among the three Basic exams: Applied Statistics, Probability, Statistics. These exams must be completed by January of the student's second year in the program. There is no penalty for attempting and not passing the third exam.

### **Statistics PhD**

Students in the Statistics PhD program must pass all three Basic exams (Applied Statistics, Probability, Statistics) at the PhD level by January of the student's second year. After completing those exams, statistics PhD students take a written exam on advanced statistics and an oral exam on statistics literature. The written exam has two versions, one covering advanced probability theory and statistics and asymptotic theory, and the other covering advanced probability theory and statistics and linear models. The advanced statistics exam version I is based on advanced topics in Stat 607 and Stat 608, and topics from Stat 725. The advanced statistics exam version II is based on advanced topics in Stat 607 and Stat 608, and topics from Stat 705. The two versions are offered in alternate years depending on which of Stat 705 and Stat 725 is offered in a year.

For the literature-based oral exam, students need to choose a topic from the list of topics in the Graduate Handbook and form an exam committee of that includes the primary faculty of that topic and two secondary faculty. Students are then given references on the chosen topic to read. The exam is in the form of oral presentation and responding to questions in front of the exam committee. A student may select a non-standard exam topic, in which

case the student must have the agreement of their committee members on the topic and the reading list.

In order to take the literature-based exam, a student is responsible for forming an exam committee by the end of September for a January exam, or by the last day of spring classes for an August exam. The GPM and the SCA must be notified of the committee members and topics by that date. Decisions on passing the exam are by unanimous consent of the exam committee, to be confirmed by the GAC. After the GAC makes its determination, the student will be notified of the result via a memo from the GPD sent by the GPM. A student who does not pass will have one more chance to pass the literature-based exam. The second attempt may be on the same or a different topic.

## Mathematics PhD

Students in the Mathematics PhD program must pass three written exams by January of the student's second year. One of these exams must be Advanced Calculus/Linear Algebra, which the student is expected to begin taking in their first semester and continue taking until passed. The remaining two exams may be chosen from among Algebra, Analysis, Applied Mathematics, Stochastics, Topology. (With the approval of the GPD, Mathematics students may also take the written Advanced Statistics exam in place one of these two exams.) These correspond to the regularly offered 600-level course sequences. Students are allowed to take up to three exams during any exam period. There is no penalty for attempting and not passing additional exams.

After completion of the written exams, students should begin to prepare for the oral examination. This examination may be attempted at most twice and must be passed by the mid-semester date of the student's sixth semester in the program.

1. By the mid-semester date of a student's fifth semester, they must name a three person exam committee of faculty in the department. One member will be designated as the chair. It is permissible to have at most one member of the committee be a Visiting Assistant Professor.
2. The student and the committee will draw up a list of topics for the exam and agree on a date and time for examination.
  - (a) The list should be roughly equivalent to 6 credits worth of material selected from 9 credits worth of courses.

- i. Typically the 9 credits will break down as 3 credits from a second year course, 3 from a reading course with a member of the committee and 3 from either source.
  - ii. The topic list will then be further refined to reflect approximately two thirds of the material from each course.
- (b) The committee member(s) responsible for second year course material need not be the actual instructor of the course which the student took, although they should be qualified to teach it.
- (c) A template topic form can be obtained from the GPM. The completed form must be provided to the GPM at least one month before the scheduled exam date. The GPM will provide it to the GPD and all committee members for approval.

The examination itself will be governed by the following rules:

1. The exam may be a combination of a presentation by the student and questions from the committee, with the presentation portion not to exceed thirty minutes; or it may consist entirely of questions.
2. The exam length will be between ninety minutes and two hours depending on the judgment of the committee.
3. The committee will decide after the exam on a recommendation to the GAC. A passing recommendation must be a unanimous decision of the committee.
  - (a) The chair of the committee shall promptly submit a short (approximately one half to one page) review of the exam to the GAC explaining the recommendation.
  - (b) The GAC will determine whether or not to accept the recommendation of the committee. The GAC retains the ultimate authority to determine the results of the exam but expects to override the committee's recommendation only in unusual circumstances.
  - (c) The GPM will send the student a memo from the GPD with the final decision of the GAC.

## **Academic Advising**

### **Graduate Affairs Committee**

The GAC oversees the graduate program, dealing with general policies as well as individual student issues. It is chaired by the GPD, and consists

of at least three more faculty members together with a graduate student representative. The graduate student representative is to be selected by the graduate students at their own discretion and approved by the GPD.

### **Academic Advisors**

The GPD serves as the academic advisor for all new mathematics PhD students and continues in this role until the student picks a dissertation advisor. During these years the student is encouraged to consult with other faculty potentially closer to their eventual research area as well. Once the student has chosen a dissertation advisor, that faculty member also serves as the academic advisor. The SCA serves the corresponding role for all new Statistics PhD students and MS students at the Amherst campus, while the SCN does so for all Statistics MS students at the Newton campus. The AMD serves as the academic advisor for students in the Applied Mathematics MS program.

The role of the academic advisor is to guide the student through the degree requirements, to help the student organize a reasonable program of courses, to help the student select an oral examination committee (when appropriate) and to provide some career guidance. During each semester there is an advising week, when students must consult with their academic advisors in order to preregister for the following semester. But students should seek advice from their academic advisors and other faculty members whenever an issue arises (or even before it arises). Ultimately, of course, it is the student's own responsibility to meet the requirements of the graduate program.

Doctoral students who have completed their exams are encouraged to choose a dissertation advisor and to form their dissertation committee as soon as practical. (See the section on Statute of Limitations for the required timeline.) They will guide the student and recommend additional courses or areas of study.

### **Policy on Conflicts Between Students and Advisors**

Although the Department strives to be a collegial environment, it is inevitable that occasionally some conflicts will develop. Very few of these rise to the level that any intervention is needed from the GAC, but one type of potential conflict is problematic enough to warrant a specific policy: a conflict between a graduate student and their advisor. (The primary case of interest here is between an advanced PhD student and their dissertation

advisor; however, these policies also apply to conflicts between a beginning PhD student or a MS student and their academic advisor.) If such a conflict reaches a level where either party feels that intervention is needed, they should schedule a meeting with the GPD to discuss the issues. (If the conflict directly involves the GPD, or if the student is concerned that the GPD is too close to the involved faculty member, then the student has the option to request that the a more distant member of the GAC plays the role of the GPD.) The GPD will then meet individually with all parties to gather information on the situation. If possible, the GPD will then meet with the parties together to attempt a mediation of the situation.

If any party is not satisfied with the resolution at this point, they may choose to present their case to the full GAC. A meeting of the GAC will then be scheduled at which all parties will be given a reasonable amount of time to present their case. At the conclusion, the GAC will deliberate and reach a final resolution within the Department.

We emphasize that the graduate student is in no way bound to a dissertation advisor, and that if the student feels the relationship is not working out, they always have the right to seek a new dissertation advisor. Obviously such a significant change can potentially impact the student's graduation time. The GAC will take into account any relevant information in its reappointment and support decisions, but makes no guarantee that extra funding will be available. Furthermore, none of these policies supersede the graduate student's right to request intervention from the Graduate School.

## **Financial Support**

### **Admissions and Appointment Procedures**

Almost all successful PhD applicants and some successful MS applicants are offered Teaching Assistantships. New TAs are chosen by the Graduate Admissions Committee. After the application deadline, committee members rate each of the applicants on the basis of their transcripts, exam scores (when appropriate), personal essays, letters of recommendation, and experience. Those applicants whom the committee decides to accept are then sent letters of acceptance and are given until April 15 to decide. If by April 15 not all available TA positions have been filled, then additional applicants whom the committee decides to accept are notified by telephone, email, and letter. The department is committed to developing and supporting a diverse and inclusive community of graduate students and makes every effort to recruit members of historically underrepresented groups in the Mathematical

Sciences.

## Teaching Assistantships

Financial support for graduate students is usually in the form of a Teaching Assistantship (TA). Teaching Assistants perform services for the Department amounting to 20 hours per week for PhD students and 15 hours per week for MS students. The duties can involve grading, working with students at drop-in centers, running recitation sections for a large lecture or multi-section course, or (more often in the case of advanced students) teaching a section of an undergraduate course.

Students teaching one discussion section are expected to hold at least two office hours per week, while students teaching two discussion sections are expected to hold at least three office hours per week.

Teaching Assistants are normally required to register for at least 9 credits each semester, with exceptions as detailed in the Coursework section above.

## Reappointment Procedures

Early in the spring semester, each eligible student is asked to indicate whether they wish to be considered for reappointment. Decisions concerning the reappointment of Teaching Assistants will be made by the GPD and the faculty members of the GAC in consultation with the Department Head. TAs who receive reappointment will be notified in writing by the Department Head by the middle of April under normal circumstances.

Teaching Assistants whose teaching is satisfactory and who are making satisfactory progress toward their degree, as defined below, can expect that their assistantship will be renewed for the normal period of residence. The normal period of residence is two years for MS students and five years for PhD students.

Renewal decisions will depend on the student's academic and teaching performance. Because all renewals are subject to the competitive aspects generated by the number of positions available and the number of people requesting support, it would be impossible to write a set of necessary and sufficient conditions for a TA to be reappointed. However, students should keep in mind the following guidelines.

1. The Department does not normally support a MS student longer than two years. Under normal circumstances a PhD student is guaranteed support for five years assuming successful and timely completion of all requirements and progress towards a dissertation. (Exceptions will

be noted in the student's admissions letter.) A PhD student may, if necessary, request support for a sixth year of support. Sixth year funding decisions are decided by the GAC based on the student's progress towards a dissertation and success in teaching. Sixth year funding always involves additional teaching duties.

2. Students who have not passed their examinations within the prescribed time limit should not expect to receive financial support from the Department or be permitted to continue in the program.
3. The academic progress of first-year students will be measured by their performance in the Fall semester courses and the evaluations from their instructors. If this performance is unsatisfactory (e.g., the student's grade point average is below B, or the student has not taken at least two courses in the Department), the GAC may decide to delay a decision on reappointment until after receiving mid-semester reports from the courses the student is taking in the Spring semester.
4. A student's support will not be renewed if their teaching performance is unsatisfactory.

### **Satisfactory progress toward the degree**

The progress toward the degree is measured differently depending on whether the student is a PhD or an MS student and the number of years the student has spent in the program.

The academic progress of first-year students in either the PhD or MS program is determined as in (3) above.

Second-year Mathematics PhD students are expected to have attempted the Advanced Calculus/Linear algebra exam and two other written exams by August preceding their second year and have passed both by January of their second year. Second-year Statistics PhD students are expected to have attempted three Basic Exams by August preceding their second year and have passed all three at the PhD level by January of their second year.

Third-year Mathematics PhD students are expected to have formed an oral examination committee during the fall semester and to have passed their oral examination by the mid-semester date of the spring semester. Third-year Statistics PhD students are expected to have attempted two Advanced Exams by August preceding their third year and have passed them by January of their third year.

Once PhD students have completed all examination requirements, they are expected to choose a dissertation advisor. With the advisor's guidance, they then must declare a dissertation Committee and submit a dissertation Proposal as described in the Statute of Limitations section.

The academic progress of PhD students in their fourth and fifth year shall be measured by their progress in their dissertation research as reported by the dissertation advisor to the GPD.

### **Spring Semester Admissions**

On rare occasions students may be accepted into the graduate program beginning in the Spring Semester. Normally, time limits for passing the qualifying exams and limits on financial support will be applied as if the student had entered the program in the previous Fall Semester.

### **Research Assistantships**

A limited number of Research Assistantships, funded by external grants, are available through some of the faculty members of the Department. Appointments and re-appointments are decided by the Principal Investigator in consultation with the GPD and the Department Head. A Research Assistant who is not offered reappointment as an RA is eligible for a TA appointment according to the guidelines detailed above.

### **Outside Sources of Funding**

A very limited number of grants and fellowships are available from various government and private agencies. These are usually awarded on a highly competitive basis, and are in some cases targeted to particular groups, e.g., women or minorities. Advanced graduate students with a proven record of achievement should especially seek out such awards. Announcements are usually posted on the Graduate Program bulletin board in the mail room; for more information, see the GPD.

### **Summer Support**

There are some opportunities for summer teaching (through the Division of Continuing Education), though the availability of such positions varies from year to year. The Department has specific guidelines establishing the priorities for filling of these positions. Students will be asked to make requests for summer teaching during the spring semester and decisions will be made

by the GPD as soon as practical. Please note that the Department is not able to provide summer funding for all interested students; students in need of summer funding should consider pursuing external sources as well.

The department attempts to make available a limited number of Departmental Research Assistantship's each summer, funded by general department funds. Only PhD students with declared dissertation advisors are eligible for these RAs. Interested students should contact the GPD, and the student recipients will be determined by the GAC based on student achievement and time in the program. Except in unusual circumstances, each student may receive a Departmental RA at most once.

Summer research support for advanced PhD students may also be available through research grants held by individual faculty members; these sources also vary from year to year.

## Department Information

Students and faculty have mailboxes in the mail room (1623D). Various important notices are distributed by a mailing list to students' departmental email accounts. Students should make sure to check these accounts regularly, or to have the mail forwarded to an account which they do check regularly. The bulletin boards in the mail room (including one for graduate student information) should be consulted regularly for notices of colloquia and seminars, sign-up sheets for qualifying exams, etc.

The department's Web site: <http://www.math.umass.edu> contains updated information about courses and other activities in the department, as well as links to many sites of interest to students and faculty. There is online access to the AMS employment listing and the Chronicle of Higher Education. Students should be on the lookout for notices of internships or summer positions in government or industry. The Campus Career Network can also be consulted.

## Seminars and Colloquia

Not everything is learned in regular courses. The Department has several distinguished lectures and colloquia each semester on Thursday afternoons at 4:00. Our location makes it possible to invite a wide variety of well-known mathematicians and statisticians to visit and speak. Graduate students should be able to benefit from many of these lectures, though some are more advanced than others.

The Connecticut Valley Mathematics Colloquium is an old tradition, shared by Amherst College, Mt. Holyoke College, Smith College and the University. Once a year each of the schools hosts a lecture by an eminent speaker, aimed at a general audience (and followed by a dinner).

Each semester a number of seminars are organized by faculty or graduate students, meeting once or twice a week. In recent years graduate students have organized the Graduate Student Seminar (GRASS) and “The what is...? Graduate Seminar” (TWIGS), featuring talks by students and faculty, respectively, about basic notions of current mathematical research at a level suitable for non-specialist graduate students.

Students often have an opportunity to talk in other seminars. Students should consult the Department Calendar to view ongoing seminars and activities. There is usually a Teaching Seminar in the Fall semester required for students preparing to teach their own discussions.

## **Centers and Special Facilities**

### **Center for Applied Mathematics and Computation**

The Center for Applied Mathematics and Computation involves faculty members and graduate students in the Department of Mathematics and Statistics who are interested in research and instruction in applied mathematics and scientific computation. The Center supports a research seminar series: Applied Analysis and Computation Seminar.

### **Research Computing Facility (RCF)**

The Research Computing Facility (RCF) provides computing facilities for the department. These facilities may be used by faculty and students as a tool for research and academic purposes. The facility is mainly UNIX and Linux based. All of the systems are networked, and support connections to the campus network and the Internet. A full-time staff and several graduate students provide support and consulting services to the community.

### **Statistical Consultation and Collaboration Services**

Statistical Consultation and Collaboration Services (SCCS) provides statistical consulting and collaboration to researchers at UMass, the Five Colleges, and elsewhere, through individual meetings and support for interdisciplinary research projects. Among other things, the center provides an opportunity

for graduate students to complement traditional studies with exposure to real world problems involving the statistical analysis of data.

## Topics Lists for Qualifying Examinations

### Advanced Calculus/Linear Algebra

This exam covers topics at the undergraduate level, most of which might be encountered in courses here such as Math 233, 235, 425, 523, 545. Faculty members who teach these courses can recommend texts for review purposes. The emphasis is on understanding basic concepts, rather than performing routine computations. But exam questions often center on concrete examples of matrices, functions, series, etc.

- Vector spaces: subspaces, linear independence, basis, dimension.
- Linear transformations and matrices: kernel and image, rank and nullity, transpose.
- Linear operators: change of basis and similarity, trace and determinant, eigenvalues and eigenvectors, characteristic polynomial, diagonalizable operators.
- Inner product spaces: orthonormal basis, orthogonal complements and projections, orthogonal matrices, diagonalization of symmetric matrices.
- Functions of one real variable: continuity and uniform continuity, derivative and Mean Value Theorem, Riemann integral, improper integrals, Fundamental Theorem of Calculus.
- Sequences and series of numbers or functions: pointwise, uniform, absolute convergence; term-by-term differentiation and integration of series; Taylor's Theorem with remainder.
- Functions of several variables: continuity, partial and directional derivatives, differentiability, maps from  $\mathbf{R}^n$  to  $\mathbf{R}^m$ , Jacobian, implicit and inverse function theorems, chain rule.
- Extrema of functions of several variables: constrained extrema, Lagrange multipliers.
- Multiple and iterated integrals, change of variables formula.
- Vector calculus: gradient, divergence, curl; line and surface integrals; theorems of Green, Gauss, Stokes; conservative vector fields.

## Probability

- Probability density and distribution functions.
- Random variables and vectors, expectation, moments.
- Joint and marginal distributions.
- Conditional distributions and expectations.
- Transformations of random variables.
- Moment generating functions.
- Independence, laws of large numbers, central limit theorems.
- Special distributions (e.g., binomial, Poisson, normal, t, F, etc.).
- Basic combinatorics.

## References

Chung, *Elementary Probability Theory*  
Woodroffe, *Probability with Applications*  
Arnold, *Mathematical Statistics*  
Casella and Berger, *Statistical Inference*

## Statistics

- Sampling distributions.
- Exponential families.
- Sufficiency and completeness.
- Estimation: maximum likelihood, method of moments, unbiasedness, efficiency, consistency.
- Bayes estimators.
- Interval estimation.
- Hypothesis testing: basic framework, UMP tests, likelihood ratio tests.

## References

Bickel and Doksum, *Mathematical Statistics*

Mood, Graybill and Boes, *Introduction to the Theory of Statistics*

Arnold, *Mathematical Statistics*

Casella and Berger, *Statistical Inference*

DeGroot, *Probability and Statistics*, 2nd ed.

## Algebra

### I. Group Theory

- Group actions; counting with groups.
- $p$ -groups and Sylow theorems.
- Composition series; Jordan-Holder theorem; solvable groups.
- Automorphisms; semi-direct products.
- Structure of finitely generated Abelian groups

### II. Linear Algebra and Commutative Algebra.

- Euclidean domain implies PID implies UFD.
- Gauss Lemma; Eisenstein's Criterion.
- Exact sequences; isomorphism theorems for modules.
- Free modules.
- Hom and tensor product of vector space, Abelian groups, and modules; Restriction and extension of scalars.
- Bilinear forms; symmetric and alternating forms; symmetric and exterior algebras.
- Structure Theorem for finitely generated modules of a PID.
- Rational canonical form.
- Jordan canonical form.
- Chain conditions; Noetherian rings and modules; Hilbert's Basis Theorem.
- Prime and maximal ideals.
- Field of fractions.
- Localization of rings and modules: exactness of localization; local rings; Nakayama's Lemma.
- Integral extensions.
- Noether's Normalization Lemma.
- Integral closure.
- Nullstellensatz.
- Closed affine algebraic sets.

### III. Field Theory and Galois Theory

- Algebraic field extensions: finite extensions; degree of extensions; the minimal polynomial; adjoining roots of polynomials; the splitting field; algebraic closure.
- Separable extensions.
- Theorem of the primitive element.
- Galois extensions: Fundamental Theorem of Galois Theory.
- Finite fields and their Galois groups; Frobenius endomorphism.
- Cyclotomic polynomial, Cyclotomic fields and their Galois groups.
- Cyclic extensions.
- Solvable extensions; Solving polynomial equations in radicals.
- Transcendence degree.

#### References

Dummit and Foote, *Abstract Algebra*

Atiyah and MacDonald, *Introduction to Commutative Algebra*

Lang, *Algebra*

## Analysis

- **Lebesgue measure:** Construction of Lebesgue measure on  $\mathbf{R}$  and  $\mathbf{R}^d$ . Measurable and non measurable sets; Cantor sets. Lebesgue and Borel measurable functions; Egorov theorem and Lusin Theorems. Construction and properties of Lebesgue integral; the space  $L^1$  of integrable functions and its completeness; comparison with Riemann integral. Fubini-Tonelli theorem in  $\mathbf{R}^d$ . Modes of convergence: convergence almost everywhere, convergence in measure, convergence in  $L^1$ .
- **Integration and differentiation:** Differentiation of the integral; Hardy-Littlewood maximal function; Lebesgue differentiation theorem. Functions of bounded variation; absolutely continuous functions; the fundamental theorem of calculus.
- **Hilbert spaces** Abstract Hilbert spaces and examples;  $L^2$  spaces; Bessel's inequality and Parseval's identity; Riemann-Lebesgue Lemma; Orthogonality; orthogonal projections. Linear transformations; linear functionals; Riesz representation theorem; adjoints transformations.
- **Fourier analysis:** Fourier transform in  $L^1$  and  $L^2$ ; Fourier inversion formula. Fourier series; Dirichlet's Theorem and Fejér's Theorem
- **General theory of measure and integration** Measure spaces and  $\sigma$ -algebras.  $\sigma$ -finite measures. Caratheodory theorem and the construction of measures; outer measures and extension theorems. Integration theory. Product measures and Fubini-Tonelli theorem. Signed measure; Radon-Nikodym theorem; Borel measures on  $\mathbf{R}$  and Lebesgue-Stieljes integral.
- **Banach spaces and  $L^p$  spaces.** Abstract Banach spaces and examples; completeness criterion. Convexity;  $L^p$ -norms; Schwarz, Hölder, Minkowski, and Jensen inequalities.  $L^p$ -spaces and their duals; Riesz-Fischer Theorem.

## References

Berberian, *Introduction to Hilbert Spaces*  
Folland, *Real Analysis*  
Gelbaum and Olmsted, *Counterexamples in Analysis*  
Halmos, *Measure Theory*

Royden, *Real Analysis*  
Rudin, *Principles of Mathematical Analysis*  
Rudin, *Real and Complex Analysis*  
Stein and Shakarchi, *Real Analysis*  
Wheeden and Zygmund, *Measure and Integral*

## Stochastics

### Part A: Probability

- Probability axioms, conditional probability and independence and probability on countable sets.
- Construction of probability measures, in particular on  $\mathbb{R}$  and  $\mathbb{R}^d$ .
- Random variables and theory of integration.
- Independence random variables and sum of independent random variables.
- Convergence of random variables (almost sure, in probability,  $L^p$ ) and laws of large numbers.
- Weak convergence and central limit theorems.
- Conditional expectation and martingales.

### Part B: Stochastics

- Simulation of random variable and Monte-Carlo methods.
- Finite and countable state space Markov chains, stationary distribution, convergence, recurrence and transient behavior. Monte-Carlo Markov chains.
- Continuous time Markov chains, poisson processes and queueing processes.
- Martingales.
- Random walks and Brownian motion.

### References (Probability)

Cinlar, *Probability and Stochastics*  
Dudley, *Real Analysis and Probability*  
Durrett, *Probability: Theory and Examples*  
Jacod and Protter, *Probability Essentials*  
Resnick, *Probability Path*  
Rosenthal, *A first look at rigorous probability*  
Shiryaev, *Probability*

## References (Stochastics)

Lawler, *Introduction to Stochastic Processes*

Resnick, *Adventures in Stochastic processes*

Durrett, *Essentials of Stochastic Processes*

Ross, *Introduction to Probability Models*

Madras, *Lectures on Monte-Carlo Methods*

Rubinstein and Kroese, *Simulation and the Monte Carlo Method*

Levin, Peres and Wilmer, *Markov chain and mixing times*

Hoel, Port and Stone, *Introduction to Stochastic Processes*

Ross, *Stochastic Processes*

Karlin and Taylor, *A first course in Stochastic Processes*

## Topology

- Topology and continuity: bases, order topology, subspace topology, product topology (infinite and finite), box topology, closed sets, limit points, Hausdorff spaces, homeomorphisms, Pasting Lemma, metric spaces, uniform topology, Uniform Limit Theorem, quotient topology, open and closed maps.
- Compactness and connectedness: connectedness of  $\mathbb{R}$ , Intermediate Value Theorem, connectedness for products, path-connectedness, components and path-components, Tube Lemma, finite intersection property, uniform continuity, Heine-Borel Theorem, Lebesgue Number Lemma, sequential compactness, limit point compactness, local compactness, compactifications.
- Complete metric spaces: Cauchy sequences, equicontinuity, Ascoli Theorem (for  $\mathbb{R}^n$ ), complete and totally bounded metric spaces.
- Definition and elementary properties of homotopy; homotopy equivalences; deformation retracts.
- The definition of  $\pi_1$ ; functoriality under mappings and invariance under homotopy. The relation between  $\pi_1$  at different base points. The fundamental group of a cartesian product.
- The path lifting/homotopy lifting lemmas, their proofs, and their use in proving that  $\pi_1(S^1) \cong \mathbf{Z}$ .
- The statement of the Seifert–van Kampen theorem, and its use in computing  $\pi_1$  of various spaces, such as compact surfaces.
- Covering spaces; path and homotopy lifting theorems; classification of connected covers via subgroups of the fundamental group.
- Cell complexes,  $\Delta$ -complexes and simplicial complexes, the classification of compact surfaces.
- Singular, simplicial and cellular homology; degree of maps between spheres (and connected orientable manifolds), induced homomorphisms, homotopy invariance; reduced homology; relative homology; long exact sequences of a pair, a triple, and the Mayer-Vietoris sequence; excision; Homology with coefficients, the universal coefficients theorem; Euler characteristic.

- Simplicial, singular and cellular cohomology; the cup product; Künneth theorems; orientations, the cap product and Poincaré duality

### References

Munkres, *Topology: A First Course*, Sections 1-7, 12-29, 43, 45

Hatcher, *Algebraic Topology*, Chapters 0, 1, 2, 3.

## **Advanced Statistics: Version I**

This exam covers the theory and application of the linear model based mostly on the course content of Stat 607, 608 and 705. Topics include:

### **References**

References for basic Statistics and Probability exams

Ravishanker and Dey, *A First Course in Linear Model Theory*, Chapters 1,2,3, 5 and 6.

Graybill, *Theory and Application of the Linear Model*, Chapters 3 and 4.

Seber and Lee, *Linear Regression Analysis, 2nd Edition*, Chapters 1 and 2.

## Advanced Statistics: Version II

This exam covers materials on Probability and Mathematical Statistics based on course content in Stat 607, 608 and 725. Topics include:

Content of basic probability and statistics and statistical models; point estimation, set estimation and hypothesis testing from a frequentist's, decision theoretic and Bayesian point of view; finite sample and asymptotic techniques in a variety of (parametric/semiparametric/non-parametric) statistical models.

### References

References for basic Statistics and Probability exams

Jun Shao, *Mathematical Statistics (2nd Edition)*. Springer Series in Statistics.

T. S. Ferguson, *A Course in Large Sample Theory*. Chapman & Hall/CRC

A. W. van der Vaart, *Asymptotic Statistics*. Cambridge Series in Statistical and Probabilistic Mathematics