# Topics Lists for Qualifying Examinations

## BASIC EXAMS

### ADVANCED CALCULUS/LINEAR ALGEBRA

This part of the Basic Exam covers topics at the undergraduate level, most of which might be encountered in courses here such as Math 233, 235, 425, 523, 545. Faculty members who teach these courses can recommend texts for review purposes. The emphasis is on understanding basic concepts, rather than performing routine computations. But exam questions often center on concrete examples of matrices, functions, series, etc.

- Vector spaces: subspaces, linear independence, basis, dimension.
- Linear transformations and matrices: kernel and image, rank and nullity, transpose.
- Linear operators: change of basis and similarity, trace and determinant, eigenvalues and eigenvectors, characteristic polynomial, diagonalizable operators.
- Inner product spaces: orthonormal basis, orthogonal complements and projections, orthogonal matrices, diagonalization of symmetric matrices.
- Functions of one real variable: continuity and uniform continuity, derivative and Mean Value Theorem, Riemann integral, improper integrals, Fundamental Theorem of Calculus.
- Sequences and series of numbers or functions: pointwise, uniform, absolute convergence; term-by-term differentiation and integration of series; Taylor's Theorem with remainder.
- Functions of several variables: continuity, partial and directional derivatives, differentiabilitity, maps from $\mathbf{R}^n$ to $\mathbf{R}^m$, Jacobian, implicit and inverse function theorems, chain rule.
- Extrema of functions of several variables: constrained extrema, Lagrange multipliers.
- Multiple and iterated integrals, change of variables formula.
- Vector calculus: gradient, divergence, curl; line and surface integrals; theorems of Green, Gauss, Stokes; conservative vector fields.

## COMPLEX ANALYSIS

- Analytic functions: algebraic and geometric representation of complex numbers; elementary functions, including the exponential functions and its relatives (log, cos, sin, cosh, sinh, ...); functions defined by power series; concept of holomorphic (analytic) function, complex derivative and the Cauchy–Riemann equations; harmonic functions.

- Complex integration: complex contour integrals, the Cauchy Theorem and the Cauchy Integral Formula; local properties of analytic functions, Taylor series expansions and their convergence; isolated singularities and Laurent series expansions. Liouville's Theorem and the Fundamental Theorem of Algebra. Maximum principle and Schwarz's lemma.

- Calculus of residues: meromorphic functions, the Residue Theorem, calculation of definite integrals by the evaluation of residues, including improper integrals (principal values) and integrands with branch points; the argument principle (for counting zeroes and poles) and the Rouche Theorem.

- Conformal mapping: geometrical interpretation of an analytic function; explicit mappings defined by elementary functions; linear fractional (bilinear) transformations and their action on the Riemann sphere; the Riemann Mapping Theorem (statement); solution of specific problems in potential theory (boundary-value problems for harmonic functions) by the conformal mapping technique.

## REFERENCES


L.V. Ahlfors, *Complex Analysis*

J.B. Conway, *Functions of One Complex Variable*

R. Greene and S. Krantz, *Function Theory of One Complex Variable*

K. Knopp, *The Theory of Functions*

S. Lang, *Complex Analysis*

J. Marsden and M. Hoffman, *Basic Complex Analysis*

E. Stein and R. Shakarchi, *Complex Analysis*

## NUMERICS

- Computer representation of numbers, and error propagation.
- Solving linear systems: direct methods—Gaussian elimination, matrix decomposition (LU), Cholesky's method; iterative methods—Jacobi, Gauss–Seidel, SOR; condition number and norms.
- Solving nonlinear equations: bisection, Newton's method, secant, fixed point methods, nonlinear systems.
- Interpolation and approximation: polynomial interpolation, trigonometric interpolation, orthogonal polynomials and least squares.
- Integration and differentiation techniques: Newton–Cotes formulas and Gaussian quadrature, numerical differentiation, Richardson extrapolation.
- Ordinary differential equations: one step methods—Euler, Taylor series, Runge–Kutta; multistep methods—explicit, implicit, predictor-corrector; consistency and stability; solution of constant coefficient finite difference equations.

## REFERENCES

J. Stoer and R. Bulirsch, *Introduction to Numerical Analysis*
E. Isaacson and H. B. Keller *Analysis of Numerical Methods*
K. E. Atkinson, *An Introduction to Numerical Analysis*
H. R. Schwarz, *Numerical Analysis: A Comprehensive Introduction*
S. Conte and C. de Boor, *Elementary Numerical Analysis*

**PROBABILITY**

- Probability density and distribution functions. Random variables and vectors, expectation, moments. Joint and marginal distributions. Conditional distributions and expectations. Transformations of random variables. Moment generating functions. Independence, laws of large numbers, central limit theorems. Special distributions (e.g., binomial, Poisson, normal, t, F, etc.). Basic combinatorics.

**REFERENCES**

Chung, *Elementary Probability Theory*
Woodroofe, *Probability with Applications*
Arnold, *Mathematical Statistics*
Casella and Berger, *Statistical Inference*

**STATISTICS**

- Sampling distributions. Exponential families. Sufficiency and completeness. Estimation: maximum likelihood, method of moments, unbiasedness, efficiency, consistency. Bayes estimators. Interval estimation. Hypothesis testing: basic framework, UMP tests, likelihood ratio tests.

**REFERENCES**

Bickel and Doksum, *Mathematical Statistics*
Mood, Graybill and Boes, *Introduction to the Theory of Statistics*
Arnold, *Mathematical Statistics*
Casella and Berger, *Statistical Inference*.
DeGroot, *Probability and Statistics*, 2nd ed.

**APPLIED STATISTICS**

Simple and multiple linear regression; correlation; the use of dummy variables; residuals analysis and diagnostics assessment of model assumptions; model building/variable selection, regression models and methods in matrix form; an introduction to weighted least squares, regression with correlated errors, generalized linear models and nonlinear regression. R functions, objects, data structures, flow control, input and output, debugging, logical design and abstraction, simulations, parallel data analyses, optimization, large data set handling, commenting and organizing code.

**REFERENCES**

Kutner, Nachtshem and Neter, *Applied Linear Regression Models*, 4th edition

Kutner, Nachtshem, Neter and Li, *Applied Linear Statistical Models, Part I-Part III*, 5th edition

Norman Matloff, *The Art of R Programming: A Tour of Statistical Software Design*

Phil Spector, *Data Manipulation with R* Paul Teetor, *The R Cookbook*

**TOPOLOGY**

- Topology and continuity: bases, order topology, subspace topology, product topology (infinite and finite), box topology, closed sets, limit points, Hausdorff spaces, homeomorphisms, Pasting Lemma, metric spaces, uniform topology, Uniform Limit Theorem, quotient topology, open and closed maps.

- Compactness and connectedness: connectedness of $\mathbb{R}$, Intermediate Value Theorem, connectedness for products, path-connectedness, components and path-components, Tube Lemma, finite intersection property, uniform continuity, Heine-Borel Theorem, Lebesgue Number Lemma, sequential compactness, limit point compactness, local compactness, compactifications.

- Complete metric spaces: Cauchy sequences, equicontinuity, Ascoli Theorem (for $\mathbb{R}^n$), complete and totally bounded metric spaces.

- Fundamental group ($\pi_1$): Basic group theory (homomorphisms, quotient groups, free products of groups), homotopy, simple-connectedness, $\pi_1$ of the $n$-sphere, $\pi_1$ of a product, deformation retract, Van Kampen Theorem, $\pi_1$ of cell complexes.

**REFERENCES**

Munkres, *Topology: A First Course*, Sections 1-7, 12-29, 43, 45
Hatcher, *Algebraic Topology*, Sections 1.1-1.2.

**M.S. IN APPLIED MATHEMATICS**

- Solutions of linear ODE with constant coefficients; flows in one and two dimensions; linear stability analysis; existence and uniqueness for initial value problems; boundary value and eigenvalue problems for linear ODE; Sturm-Liouville theory and eigenfunction expansions.

- Basic linear PDE; evolution equations in one space variable; the wave equation (D'Alembert's formula), the diffusion (heat) equation; equilibrium equations in two and three dimensions: the Laplace and Poisson equation in simple geometries; separation of variable methods, Fourier series and transform methods.

**REFERENCES**

Blanchard, Devaney and Hall, *Differential Equations*

Boyce and DiPrima, *Elementary Differential Equations and Boundary Value Problems*

Churchill and Brown, *Fourier Series and Boundary Value Problems*

Haberman, *Elementary Applied Partial Differential Equations*

Hirsch and Smale, *Differential Equations, Dynamical Systems and Linear Algebra*

Pinsky, *Differential Equations and Boundary Value Problems with Applications*

Strauss, *Partial Differential Equations: An Introduction*

Strogatz, *Nonlinear Dynamics and Chaos*

Weinberger, *A First Course in Partial Differential Equations*