Answer all seven questions. Justify your answers.
Passing standard: 70% with four questions essentially complete.

1. Let $X, Y$ be topological spaces. Prove the following:
   
   (a) For any subset $A \subset X$, we have $X \setminus A = X \setminus \text{Int}(A)$.
   
   (b) A map $f : X \rightarrow Y$ is closed $\iff f(\overline{A}) \supset f(A)$ for all $A \subset X$.
   
   (Recall that a map is closed if it maps closed sets to closed sets.)

2. A topological space $X$ is normal if points are closed, and, for any pair of disjoint closed sets $A, B \subset X$, there are disjoint open sets $U, V \subset X$ with $A \subset U$ and $B \subset V$. Let $Y$ be a closed subspace of a normal space $X$. Show that $Y$ and the quotient $X/Y$ are normal.

3. Let $M = M_1 \times M_2$, where $M_i$ is a topological manifold of dimension $m_i$, for each $i = 1, 2$. Prove that $M$ is also a topological manifold, and moreover if each $M_i$ is
   
   (a) compact
   
   (b) connected

   then so is $M$, in each case.

4. Find all connected covering spaces of $X = \mathbb{RP}^2 \vee S^1$ of covering degree $\leq 3$. Tell which coverings are normal (i.e. regular).

5. Let $M$ be the compact surface, obtained as the quotient space of an octagon, whose edges are identified by labeling them (say, clockwise) as $aaxbyb^{-1}y^{-1}$. Compute $\pi_1(M)$, and the homology and cohomology groups $H_i(M; G), H^i(M; G)$, for $G = \mathbb{Z}, \mathbb{Z}_2, \mathbb{Z}_3$. Tell which standard surface $M$ is homeomorphic to. (You can use the classification of surfaces.)

6. Let $M = \mathbb{RP}^3 \# S^1 \times S^2$, i.e. connected sum of these 3–manifolds. Calculate the fundamental group and the integral homology groups of $M$.

7. Assuming that the complex projective plane $\mathbb{CP}^2$ is a finite simplicial complex, prove that any continuous map $f : \mathbb{CP}^2 \rightarrow \mathbb{CP}^2$ has a fixed point, i.e. there exist $x \in \mathbb{CP}^2$ such that $f(x) = x$. 