Answer all seven questions. Justify your answers.
Passing standard: 70% with four questions essentially complete.

1. Let $F : X \to Y$ be a continuous bijection, where $X$ is compact and $Y$ is Hausdorff. Show that $F$ is homeomorphism.

2. Let $CX$ be the cone over the topological space $X$, i.e. $CX = ([0,1] \times X)/\sim$, where $0 \times x \sim 0 \times x'$ for every $x,x' \in X$. Prove that
   (a) $CX$ is path-connected
   (b) $CX$ is compact if $X$ is
   (c) $CX$ is locally connected if and only if $X$ is.

3. Find all 2 and 3-sheeted covers of $S^1 \lor S^1$ up to isomorphism of covering spaces without base point.

4. Show that any map $S^2 \to S^1$ is null homotopic. Then show, by an example, that the same does not hold for $T^2 \to S^1$.

5. $X \subset \mathbb{R}^3$ is the union of the 3 coordinate lines through the origin $O$. Let $Z = \mathbb{R}^3 \setminus X$.
   (a) Compute $\pi_1(Z)$.
   (b) Compute $H_i(Z,Z \setminus O)$ for all $i$.

6. Let $X$ be the quotient $(S^1 \times I)/\sim$ by the following equivalence relation $\sim$:
   $\bullet$ $(z,0) \sim (w,0)$ if and only if $z^4 = w^4$
   $\bullet$ $(z,1) \sim (w,1)$ if and only if $z^{10} = w^{10}$
   $\bullet$ in all other cases, $(z,s) \sim (w,t)$ only when $z = w$ and $s = t$.
   Compute $H_i(X;G)$ and $H^i(X;G)$, for $G = \mathbb{Z}$ and $\mathbb{Z}_2$, respectively.

7. Prove that for any compact connected 3–manifold $M$, the Euler characteristic $\chi(M) = 0$. Recall that $\chi(M) = \sum_i (-1)^i \text{rank}(H_i(M;G))$, where $G$ can be taken as $\mathbb{Z}$ or any field.