DEPARTMENT OF MATHEMATICS AND STATISTICS
UMASS - AMHERST
BASIC EXAM - PROBABILITY
FALL 2007

Work all problems. 60 points are needed to pass at the Masters Level and 75 to pass at the Ph.D. level.

1. Suppose $X_i \overset{\text{i.i.d.}}{\sim} U(0,10), i = 1, \ldots, n$. Note that $f(x) = 1/10, 0 \leq x \leq 10$, $E(X) = 5$ and $Var(X) = 100/12$.

   (a) (5 pts) Write down an expression for the probability that all X_is are greater than 1.

   (b) (10 pts) Let $\overline{X} = n^{-1} \sum_{i=1}^{n} X_i$. Find an expression that involves $\overline{X}$ and known constants that converges to a N(0,1) distribution as n gets large. What theorem is your result based on?

   (c) (10 pts) Find the mean of $1/X_i$.

2. Let X and Y be random variables with pdf:

   $f_{X,Y}(x,y) = 1, 0 \leq x \leq 1, x \leq y \leq x+1,$ and $f_{X,Y}(x,y) = 0$ otherwise.

   (a) (5 pts) Show that $f(x,y)$ is a density.

   (b) (5 pts) Are X and Y independent? Why or why not?

   (c) (5 pts) Find $f_X(x)$.

   (d) (5 pts) Find E$(Y|X = x)$.

   (e) (5 pts) Find Pr$(X + Y < 0.5)$

3. Suppose $X \overset{\text{ind.}}{\sim} Bin(n,p)$ and $Y \overset{\text{ind.}}{\sim} Bin(m,p)$. Note that the $Bin(k,q)$ probability mass function is $\binom{k}{x} q^x (1-q)^{k-x}, x = 0, \ldots, k, 0 \leq q \leq 1$.

   (a) (15 pts) Find the conditional distribution of X given that $X + Y = j$. Give the probability mass function of this conditional distribution and identify it by its family name and parameters.

   (b) (5 pts) What is $Pr(X > Y)$?

   (c) (5 pts) What is $Pr(X/Y = 1)$?

4. Suppose $X|Y = y \sim$ Poisson(y) and $Y \sim$ Unif$(0,1)$. (The $Poisson(\lambda)$ pmf is $f(x) = \exp(-\lambda)\lambda^x/x!$ when $\lambda > 0$ and $x = 0, 1, 2, \ldots$ and zero otherwise.)

   (a) (5 pts) What is the mean of X?

   (b) (10 pts) What is the marginal distribution of X?

   (c) (10 pts) What are E(XY) and Var(XY)?