Math 331 **October 2009**

Differential Equations

Robin Young

Practice Midterm

Name: ______________________________ **Id:** ____________

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1. Find the value of the parameter k which makes the equation

$$2\,y^2 - \frac{y}{x^2} + (x^k + 4\,x\,y + 1)\,\frac{dy}{dx} = 0.$$

exact. For this value of k, find the (implicit) solution which satisfies the initial condition

$$y(1) = 1.$$

2. Consider the one-parameter family of equations

$$\frac{dy}{dt} = f_\beta(y), \quad \text{where} \quad f_\beta(y) = y - \frac{1}{3}y^3 + \beta,$$

and β is a varying parameter.

   (a) Draw the graph of f_β for $\beta = 0$, showing the local maximum and minimum.
   (b) Explain how the graph changes as β varies.
   (c) Draw phase lines and sketches of the solution for $\beta = -1$, 0 and 1. Be sure to label the equilibria.
   (d) Find the bifurcation values of β: these are the points where the number of equilibria changes.

3. A tank contains 10 kg of a solvent in 200 litres of water. A pulsating flow through the tank begins, with the inflow at time t given by $1 - \cos t$ litres per minute. The outflow equals inflow, so that the volume of solution in the tank remains constant. The concentration of solvent in the inflow pipe is 0.7 kg per litre. Assume that the tank is well mixed.

   (a) Write down a differential equation for the amount of solvent in the tank. Be sure to include the initial condition.
   (b) **Without** solving the equation, decide whether the amount of solvent settles down to a certain value after a long time, and if so, find this value. Explain your reasoning.

4. Use the following steps to solve the nonlinear Bernoulli equation

$$\frac{dy}{dt} + 3\,y = 4\,e^t\,y^2.$$

   (a) Make the substitution $v = 1/y$ and derive (and simplify) the equation satisfied by v.
   (b) Find the *general* solution of the (linear) DE for v.
   (c) Use this to give the general solution y.

5. We wish to solve the equation

$$\frac{dy}{dt} = 2\,t + y, \quad y(0) = 0$$

numerically.

(a) Write down Euler's method for doing so with a general stepsize h.

(b) Now set $h = 0.1$ and carry out three steps of your method.