Problem Set 4: Extra Problems

(1) Let $f : S^1 \to S^1$ be a continuous map which is not homotopic to the identity map. Prove that $f(x) = -x$ for some point $x \in S^1$.

(2) Let X be a path-connected space. When is it true that for any two points $p, q \in X$ all paths from p to q induce the same isomorphism between $\pi_1(X, p)$ and $\pi_1(X, q)$?

(3) Describe the homomorphism $f_* : \pi_1(S^1, 1) \to \pi_1(S^1, f(1))$ induced by each of the following maps:

  (a) The antipodal map $f(e^{i\theta}) = e^{i(\theta+\pi)}$.

  (b) The map $f(e^{i\theta}) = e^{in\theta}$ for $n \in \mathbf{Z}$.

  (c) The map $f(e^{i\theta}) = \begin{cases} e^{i\theta} & 0 \le \theta \le \pi; \\ e^{i(2\pi-\theta)} & \pi \le \theta \le 2\pi. \end{cases}$

(4) Find the fundamental group of a möbius strip.