

**MATH 300.1      WORKSHEET 2**

Chapters 2, 3 and 5:

Use the Euclidean Algorithm in the following problems.

1. Find the quotient and remainder when  $b$  is divided by  $a$ .  
(a)  $a = 11, b = -231,$       (b)  $a = -17, b = 182.$
2. Find the greatest common divisor  $\gcd(a, b)$ .  
(a)  $a = 598, b = 132,$       (b)  $a = 432, b = -282.$

Use the Extended Euclidean Algorithm in the following problems.

3. Write  $\gcd(100, 35)$  in the form  $100x + 35y$  for some  $x, y \in \mathbb{Z}$ .
4. Find one integer solution, if there is one, of the linear Diophantine equations:  
(a)  $14x + 18y = 9,$       (b)  $11x + 15y = 5.$
5. Find all integer solutions of the linear Diophantine equation  $15x - 24y = 9$ .
6. Find all positive integer solutions of the linear Diophantine equation  $20x + 25y = 200$ .
7. Solve the linear congruences:  
(a)  $4x \equiv 6 \pmod{14}$       (b)  $29x \equiv 43 \pmod{128}.$

Miscellaneous (use properties of  $\mathbb{Z}_m$ , Fermat's little, Euler-Fermat, street smart, etc.)

8. If  $2p^2 = q^3$ , where  $p, q \in \mathbb{Z}$ , show that 2 is a common divisor of  $p$  and  $q$ .
9. Show that  $\sqrt[3]{4}$  and  $1 - \sqrt{2}$  are not rational numbers.
10. Construct the addition and multiplication tables for  $\mathbb{Z}_5$ .
11. Solve for  $x \in \mathbb{Z}$ :  $x^6 \equiv 6x \pmod{7}$ .
12. Find the multiplicative inverse of  $[3]$  in  $\mathbb{Z}_{41}$ .
13. Find the remainder when  $14^{181}$  is divided by 99.

Problems involving the concepts "relation" and "equivalence relation".

14. Determine whether the following relations on  $\mathbb{Z}$  are reflexive, symmetric, or transitive.  
(a)  $aRb$  if and only if  $a + b \neq 1$ .  
(b)  $aRb$  if and only if  $a - b \geq 0$ .  
(c)  $aRb$  if and only if  $a \neq b$ .
15. Show that the following relation is an equivalence relation: for any non-zero real numbers  $a$  and  $b$ ,  $aRb$  if and only if  $ab^{-1} \in \mathbb{Q}$ .