Math551 Midterm Oct. 25, 2016

Name:____

Instructions

- **Turn off all cell phones and watch alarms!**
- There are 6 questions. Do all work in this exam booklet. You may continue work to the backs of pages, but if you do so indicate where.
- This is a "closed-book" Exam: do not use any book, calculator, or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**

QUESTION	POSSIBLE	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
TOTAL	120	

1. Consider finding a root of $f(x) = x^{1/3}$ using Newton's method with initial guess $x_0 = 1$.

   (a) Write down the Newton iteration and make a table showing the first three steps x_0, x_1, x_2.

   (b) Does the iteration converge linearly, quadratically, or some other order? If it is linear, what is the rate? Justify your answer by citing a theorem or giving direct proof.

2. Consider finding a root of $f(x) = \log(x) + 2 - x$ using the bisection method.

   (a) There is a root $x > 2$. Bracket the root using the intermediate value theorem.

   (b) Using your interval, bound the error after n steps. How many iterations would be required to find a point within 2^{-10} of the actual root?

3. Consider the matrix

$$A = \begin{bmatrix} 10^{-10} & 2 \\ 2 & 2 \end{bmatrix}$$

(a) Find the LU decomposition for A.

(b) Find the PA = LU decomposition for A.

(c) What advantages does the PA = LU decomposition have for this matrix?

4. (a) Define the forward and backward error for the solution of $A\mathbf{x} = \mathbf{b}$ with approximate solution $\mathbf{x}_a$.

(b) Compute the true solution $\mathbf{x}$ for

$$A = \begin{bmatrix} 1.01 & 1 \\ 1 & 1 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(c) For the system in part b, compute the relative forward error, relative backward error, and error magnification for the approximate solution $\mathbf{x}_a = [1, 0]^T$.

5. Consider solving $A\mathbf{x} = \mathbf{b}$ using the Gauss-Seidel Method where

$$A = \begin{bmatrix} 4 & 3 \\ 5 & 4 \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad \mathbf{x}_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

(a) Find the first three terms of the iteration, x_0, x_1, x_2.

(b) Will the iteration converge? (Hint: find the infinity norm of the iteration matrix).

6. (a) Find all cubic polynomials that pass through the points $(-1, 2), (1, 4), (2, 8)$.

(b) Find a cubic polynomial that passes through $(-1, 2), (1, 4), (2, 8), (0, 10)$.