Math551 Final Exam

Name:_____

Instructions

- Do all work in this exam booklet. You may continue work to the backs of pages, but if you do so indicate where.
- This is a "closed-book" Exam: do not use any book, calculator, or paper except this exam booklet.
- Organize your work in an unambiguous order. Show all necessary steps.
- **Answers given without supporting work may receive 0 credit!**

QUESTION	PER CENT	SCORE
1	20	
2	20	
3	20	
4	20	
5	20	
6	20	
TOTAL	120	

1. Consider a three-point difference quotient of the form

$$D(h) = \frac{af(x+2h) + bf(x+h) + cf(x)}{h}$$

used to approximate $f'(x)$. Find a, b, c to give the maximal order truncation error possible.

2. We interpolate $\sin(x)$ with a cubic polynomial at the four nodal points $x = 1, 2, 3, 4$. Give an explicit upper bound for the interpolation error at the point $x = 6$. (Your upper bound should be a number.)

3. Consider the least-squares fit of a quadtratic polynomial through the points

$$(-2,-5),(-1,1),(1,1),(2,7).$$

Compute the normal equations. Do not solve the normal equations.

4. Consider approximating $\int_0^1 f(x)\,dx$ with the quadrature rule $I_h(f) = \omega_1 f(0) + \omega_2 f(1/4) + \omega_3 f(1/2)$. Find the weights ω_1, ω_2, and ω_3 such that the degree of precision is maximal. What is the degree of precision in this case?

5. In the following table are recorded the results of applying the composite trapezoidal rule to integrate a function on the interval $[0, 1]$. Use Romberg integration to improve the estimate for the integral.

h	$I_h(f)$
1	-1
1/2	5/4
1/4	17/16

6. Write down the implicit Euler method for integrating $y' = f(t, y)$, and find the local truncation error. Compute a single step of the implicit Euler method for the equation $y' = -y^2$ with initial condition $y(0) = 1$ and step size $h = 1/4$.