STAT516 Final Spring 2017

Name:
Student ID:
Show all work. Explain your answers. Partial credit will be given.

1. The Rockwell hardness index for steel is determined by pressing a diamond point into the steel and measuring the depth of penetration. For 50 specimens of an alloy of a steel, the Rock-well hardness index averaged 62 with standard deviation 8. The manufacturer claims that this alloy has an average hardness index of at least 64.

   (a) (10pts) Is there sufficient evidence to refute the manufacturer's claim at the 1% significance level?

   (b) (10pts)The steel is sufficiently hard to meet usage requirements if the mean Rockwell hardness measure does not drop below 60. Using the rejection region found in the previous part, find the probability of a type II error, β, for the specific alternative $\mu_a = 60$.

   (c) (10pts) How large should the sample size be if we require that $\alpha = 0.01$ and $\beta = 0.05$ when $\mu_a = 60$?

2. In a study by Susan Beckham and her colleagues, measurements were made of anterior compartment pressure (in millimeters of mercury) for ten healthy runners and ten healthy cyclists. The data is summarized as follows:

	Runners		Cyclists	
Condition	Mean	s	Mean	s
Rest	14.5	3.92	11.1	3.98
80% maximal O_2 consumption	12.2	3.49	11.5	4.95

   (a) (10pts) Is there sufficient evidence to support a claim that the variability of compartment pressure differs for runners and cyclists who are resting? Use $\alpha = 0.05$.

   (b) (10pts) Is there sufficient evidence to justify claiming that a difference exists in mean compartment pressures for runners and cyclists who are resting? Use $\alpha = 0.05$. Bound or determine the associated p-value.

3. Auditors are often required to compare the audited (or current) value of an inventory item with the book (or listed) value. If a company is keeping its inventory and books up to date, there should be a strong linear relationship between the audited and book values. A company sampled ten inventory items and obtained the audited and book values given in Table 1. Table 2 contains part of the regression analysis results by R, a statistical software.

Item	Audit Value (y_i)	Book Value (x_i)
1	9	10
2	14	12
3	7	9
4	29	27
5	45	47
6	109	112
7	40	36
8	238	241
9	60	59
10	170	167

Table 1: The data

Table 2: The regression equation is $y = 0.7198 + 0.9914x$

	Estimate	Standard deviation	T-ratio	p-value
β_0	0.7198	1.1764	0.612	0.558
β_1	0.9914	0.0114	86.994	3.4e-13

(a) (10pts) Write out the formula that is used to compute the estimate of β_1, and the one that is used to compute the standard deviation of β_1.

(b) (10pts) What is your estimate for the expected change in the audited value for a one-unit change in book value? Construct a 95% CI for the expected change.

(c) (20pts) Note if $\beta_0 = 0$ and $\beta_1 = 1$, the book value and the audited value are consistent. To test if the book value and the audited value are consistent, we can simultaneously perform the following two separate hypothesis testing at $\alpha = 0.025$ respectively:

$$H_0 : \beta_0 = 0 \text{ vs } H_a : \beta_0 \neq 0$$

$$H_0 : \beta_1 = 1 \text{ vs } H_a : \beta_1 \neq 1$$

Is there sufficient evidence suggesting the book value is different from the audited value? {*Hint: If at least one of the null hypotheses is rejected, they are not consistent.*}

(d) (10pts) If the book value is $x = 100$, estimate the expected audited value and construct a 95% CI for the expected audited value. It is known that $\bar{x} = 72, S_{xx} = 54714, S = 2.666$.