STAT 516 Spring 2014 Exam 2

Name_____ **ID**_____

–SHOW YOUR WORK WHERE WORK IS REQUIRED. NO WORK, NO CREDIT!
–IF YOU DON'T KNOW HOW TO DO A PROBLEM, MOVE ON AND COME BACK TO IT
–READ THE QUESTIONS CAREFULLY!! –GOOD LUCK

1. Let $Y_1, \cdots, Y_n$ be a random sample from the uniform distribution on the interval $(\theta, \theta + 1)$. Let

$$\hat{\theta}_1 = \bar{Y} - \frac{1}{2} \text{ and } \hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}$$

where $Y_{(n)}$ is the sample maximum.

   (a) (10pts) Show that $\hat{\theta}_1$ is the method of moments estimator for θ.

   (b) (10pts) Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ.

   (c) (10pts) Show that $\hat{\theta}_1$ is consistent.

   (d) (10pts) Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.

2. Let $Y_1, \cdots, Y_n$ be a random sample from the Gamma(m, θ) distribution whose density function is given by

$$f(y|\theta) = \frac{1}{(m-1)!\theta^m} y^{m-1} e^{-y/\theta}$$

for $y > 0$. Note that $EY_1 = m\theta$ and $Var(Y_1) = m\theta^2$.

   (a) (10pts) Write out the likelihood of the sample.

   (b) (10pts) Find the MLE $\hat{\theta}$ of θ.

   (c) (10pts) Show that $\hat{\theta}$ is unbiased and find its variance. Show that the variance of $\hat{\theta}$ converges to zero as n goes to infinity, and therefore $\hat{\theta}$ is consistent.

   (d) (10pts) What is the sufficient and complete statistic for θ?

   (e) (10pts) Is $\hat{\theta}$ MVUE? Elaborate.

   (f) (10pts) Find the MLE for the variance of Y_1, which is $m\theta^2$.