STAT 516 Spring 2014 Exam 1

Name_____ **ID**_____

–SHOW YOUR WORK WHERE WORK IS REQUIRED. NO WORK, NO CREDIT!
–IF YOU DON'T KNOW HOW TO DO A PROBLEM, MOVE ON AND COME BACK TO IT
–READ THE QUESTIONS CAREFULLY!!
- SEE SEPARATE SHEET FOR TABLES. –GOOD LUCK

1. Let $Y_1, \cdots, Y_n$ be a random sample from an exponential distribution with parameter β. That is, Y_i has density function $f(y_i) = e^{-y_i/\beta}(1/\beta), y_i \geq 0, \beta > 0$ with $E(Y_i) = \beta$ and $Var(Y_i) = \beta^2$. Consider estimating β with the estimators $\hat{\beta}_1 = \bar{Y}$ and $\hat{\beta}_2 = n\bar{Y}/(n+3)$.

   (a) (10pts)Find the bias, variance and MSE of $\hat{\beta}_2$.

   (b) (10pts)Find the standard error and estimation error bound of $\hat{\beta}_1$.

   (c) (10pts)With the estimator $\hat{\beta}_1$, find the sample size necessary to estimate β to be within 0.05 with probability 0.95, if β is thought to be approximately 1.

   (d) (10pts)When n is large, what is the approximate distribution of $\hat{\beta}_1$?

   (e) (10pts)Construct a 95% confidence interval based on the approximate distribution of $\hat{\beta}_1$ from the previous part.

   (f) (10pts)Let the sample size $n = 1$. Let Y denote the only sample. Prove that Y/β is a pivot.

   (g) (10pts) Use the pivot Y/β from the previous part to find a 90% lower confidence limit for β.

2. For the following, you don't have to carry out the final calculation, but set up the answers with **all numerical values**.

   (a) (7pts)A random sample of 25 vitamin tablets are collected and the amount of vitamin C (in mg) in each tablet is measured. A sample mean of 253 and a standard deviation of 9 mg are observed for the vitamin C content. Assume that the vitamin C content from a randomly chosen tablet follows a normal distribution with mean μ and variance σ^2. Find a 90% confidence interval for the true mean vitamin C content μ.

   (b) (7pts)Using the same data and assumption from the previous part, find a 90% confidence interval for the true variance σ^2.

(c) (8pts) In order to study Massachusetts voters' opinions about same sex marriages, two independent random samples are taken, one from Hampshire and one from Franklin county. The following results are obtained:

Group	Sample size	Total in favor
Franklin	100	50
Hampshire	200	120

Give an estimate, an estimated standard error for that estimate and an approximate 90% confidence interval for the true proportion in favor in Franklin county. Be sure to identify each of the three items.

(d) (8pts) Using the same data from the previous part, give an estimate, an estimated standard error for that estimate and an approximate 95% confidence interval for the difference between the proportion in favor in Franklin and Hampshire college (defining the difference as Franklin - Hampshire). Be sure to identify each of the three items.