Name: ____________________

Stat 516
Midterm Examination
March 12, 2015
(10:00 AM - 11:15 AM)

Instructions:

- The total score is 100 points.
- There are six questions. Each one is worth 20 points.
  TA will grade the best **five** questions that you solved.
- Show **ALL** your work!!
- Some questions have more than one parts. Check carefully to ensure that you don't miss any parts.
- Don't scratch on the line marked **Score** on the bottom of each page.
- You are allowed to use one 8.5x11(letter size) double-sided formula sheet and a calculator in this exam.

1. The tensile strength for a type of wire is **normally distributed with mean μ and variance σ^2**. Six pieces of wire($n = 6$) were randomly selected from a large roll; and Y_i, the tensile strength for portion i is measured for $i = 1, 2, \ldots, 6$.

(2points) (a) What is the sampling distribution of $\bar{Y} = \frac{1}{6}(Y_1 + \cdots + Y_6)$? (**No justification required**)
[Sol]

(8points) (b) Suppose the population mean μ is unknown, but **the population variance σ^2 is known as 6**. Find the probability that $\bar{Y}$ will be within 2 of the true population mean μ.
[Sol]

(10points) (c) Suppose the population mean μ and **variance σ^2 are unknown**. Then we can estimate μ and σ^2 by $\bar{Y}$ and S^2(sample variance). Find the probability that $\bar{Y}$ will be within $2S/\sqrt{n}$ of the true population mean μ.
[Sol]

Score:____________

2. Candidate A in a city election believes that 30% of the city's voters favor him. Suppose the $n = 20$ voters from the city show up to vote. Note that we think of $n = 20$ voters as a random sample from this city.

(2points) (a) Suppose Y is the number of voters who vote him. What is the probability distribution for Y?(Hint. Y is a discrete random variable with a well-known probability distribution)
[Sol]

(8points) (b) What is the *exact* probability that candidate A will receive 25% of their votes?
(Hint. The following information will be helpful for the probability calculation: $P(Y \leq 6) = 0.608$, $P(Y \leq 5) = 0.416$, $P(Y \leq 4) = 0.238$, $P(Y \leq 3) = 0.107$).
[Sol]

(10points) (c) What is the *approximate* probability that candidate A will receive 25% of their votes?
(Hint. Use the central limit theorem and the 0.5 continuity correction).
[Sol]

 Score:____________

3. Suppose that Y_1, Y_2 and Y_3 denote a random sample from a **normal distribution with mean** $E(Y_i) = \mu$ **and** $V(Y_i) = \sigma^2$ for $i = 1, 2, 3$. Note that σ^2 is known. Consider the following two estimators of μ:

$$\hat{\mu}_1 = \frac{0.5Y_1 + 2Y_2 + 0.5Y_3}{3} \quad \text{and} \quad \hat{\mu}_2 = \bar{Y} = \frac{Y_1 + Y_2 + Y_3}{3}.$$

(10points) (a) Show that $\hat{\mu}_1$ and $\hat{\mu}_2$ are unbiased estimators for μ.

[Sol]

(10points) (b) Calculate the variances of $\hat{\mu}_1$ and $\hat{\mu}_2$, and find an estimator that has the smallest MSE(Mean Square Error).

[Sol]

 Score:____________

4. Suppose that $Y_1, \ldots, Y_n$ represent a random sample from an **exponential distribution with parameter** θ.

(4points) (a) What is the moment generating function of Y_i where $i = 1, \ldots, n$?

[Sol]

(8points) (b) What is the sampling distribution of $S = \sum_{i=1}^{n} Y_i$? (Hint : You can use the method of moment generating functions) (**Justification required**).

[Sol]

(8points) (c) Find an unbiased estimator for θ and calculate its standard error.

[Sol]

5. In polycrystalline aluminum, the number of grain nucleation sites per unit volume is modeled as having a **Poisson distribution with mean** λ. Fifty unit-volume test specimens subjected to annealing under regime A produced an average of 50 sites per unit volume. Fifty independently selected unit-volume test specimens subjected to annealing regime B produced an average of 65 sites per unit volume.

(10points) (a) Estimate the mean number λ_A of nucleation for regime A and place a 2-standard error bound on the error of estimation.

[Sol]

(10points) (b) Estimate the difference in the mean numbers of nucleation sites $\lambda_A - \lambda_B$ for regime A and B. Place a 2-standard error bound on the error of estimation.

[Sol]

 Score:____________

6. The administrator for a hospital wished to estimate the average number of days (μ) required for inpatient treatment of patients between the ages of 25 and 34. A random sample of 100($=n$) hospital patients between these ages produced a sample mean and sample variance equal to 5.4($=\bar{y}$) and 3.1($=s^2$), respectively.

(5points) (a) Find an unbiased estimator for the mean length of stay for the population of patients from which the sample was drawn, μ, and calculate its standard error.

[Sol]

(15points) (b) Construct a 95% two-sided confidence interval for the mean length of stay for the population of patients (Hint: Use a large-sample confidence interval formula.)

[Sol]

 Score:____________