

Name: _____

STAT 516
FINAL EXAM
MAY 4, 2015
(8:00 AM - 10:00 AM)

Instructions:

- The total score is 120 points.
- There are six questions. Each one is worth 20 points.
- Show **ALL** your work!!
- Some questions have more than one parts. Check carefully to ensure that you don't miss any parts.
- Don't scratch on the line marked **Score** on the bottom of each page.
- You are allowed to use two 8.5x11(letter size) double-sided formula sheets and a calculator in this exam.

1. Suppose that $Y_1, Y_2, \dots, Y_n (n \geq 3)$ denote a random sample from a distribution with $E(Y_i) = p$ and $V(Y_i) = p(1-p)$ for all i . Consider the following two estimators of unknown parameter p :

$$\hat{p}_1 = \frac{(n-1)Y_{n-1} + Y_n}{n}, \quad \hat{p}_2 = \bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i.$$

- (6points) (a) Calculate the biases of \hat{p}_1 and \hat{p}_2 .

[Sol]

- (8points) (b) Calculate the variances of \hat{p}_1 and \hat{p}_2 , and show that \hat{p}_2 is a superior estimator by finding the efficiency of \hat{p}_2 relative to \hat{p}_1 .

[Sol]

- (6points) (c) Are \hat{p}_1 and \hat{p}_2 consistent estimators for p ? (**Justification required**).

[Sol]

2. Suppose that Y_1, \dots, Y_n represent a random sample from the exponential distribution given by

$$f(y) = \frac{1}{\beta} e^{-\frac{y}{\beta}}$$

where $y > 0, \beta > 0$.

- (7points) (a) Find the MVUE(Minimum Variance Unbiased Estimator) for β (**Show all your work**).
[Sol]

- (6points) (b) Find the method-of-moments estimator for β (**Show all your work**).
[Sol]

- (7points) (c) Find the MLE(Maximum Likelihood Estimator) for β (**Show all your work**).
[Sol]

3. The output voltage for an electric circuit is specified to be 130. A sample of 40 independent reading on the voltage for this circuit gave a sample mean 128.6 and a sample standard deviation 3.9.

(10points)

- (a) Is there sufficient evidence to indicate that the average output voltage, μ is less than 130? Use a **large-sample test** with significance level $\alpha = .05$.

[Sol]

(10points)

- (b) Calculate associated p-value in (a). What can be said about the calculated p-value?

[Sol]

4. Chronic anterior compartment syndrome is a condition characterized by exercise-induced pain in the lower leg. Swelling and impaired nerve and muscle function also accompany the pain, which is relieved by rest. Researchers conducted an experiment involving 25 healthy runners and 20 healthy cyclists to determine if pressure measurements within the anterior muscle compartment differ between runners and cyclists. The data - compartment pressure, in millimeters of mercury - are summarized in the following table :

Condition	Runners		Cyclists	
	\bar{x}	s_1	\bar{y}	s_2
Rest	12.2	3.49	11.5	4.95
Maximal O_2 consumption	19.1	16.9	12.2	4.67

Assume that pressure measurements within the anterior muscle compartment are **normally distributed**, and that **the variances of pressure measurements are equal** for runners and cyclists, and that the samples are independent.

- (8points) (a) Construct a 95% confidence interval for the difference in mean compartment pressures between runners and cyclists under the resting condition.

[Sol]

- (8points) (b) Construct a 95% confidence interval for the difference in mean compartment pressures between runners and cyclists who exercise at maximal O_2 consumption.

[Sol]

- (4points) (c) What can you conclude from the intervals you obtained in (a) and (b)?

[Sol]

5. Question 4 presented some data concerning chronic anterior compartment syndrome.

(10points)

- (a) Is there sufficient evidence to justify claiming that a difference exists in mean compartment pressures between runners and cyclists under the resting condition? Use $\alpha = 0.1$. Bound or determine the associated p-value.

[Sol]

(10points)

- (b) Do the data provide sufficient information to indicate that the variability of compartment pressure for runners at maximal O_2 consumption is larger than that of compartment pressure for cyclists at maximal O_2 consumption? Use $\alpha = 0.05$. Bound or determine the associated p-value.

[Sol]

6. Suppose that X_1, \dots, X_n denote a random sample from a Poisson distribution with unknown mean λ .

(7points)

- (a) Find the MLE of the parameter λ (**Show all your work**).

[Sol]

(6points)

- (b) Is the estimator that you found in (a) a consistent estimator for λ ? (**Justification required**)

[Sol]

(7points)

- (c) Find the approximate distribution of $\frac{\bar{X}-\lambda}{\sqrt{\bar{X}/n}}$ for large n (Hint : the Central Limit Theorem says that the approximate distribution of $U_n = \frac{\bar{X}-\lambda}{\sqrt{\lambda/n}}$ is a standard normal distribution for large n) (**Show all your work**).

[Sol]