STAT516 Practice Final Spring 2016

Name:
Student ID:
Show all work. Explain your answers. Partial credit will be given.

1. Assume a sample of size 16 is drawn from a normal distribution with mean μ and standard deviation σ. Assume that $\sigma = 4$. Consider testing the hypothesis $H_0 : \mu = 0$ versus $H_1 : \mu \neq 0$.
   (a) Let the rejection region be $|\bar{X}| > 2$ where $\bar{X}$ is the sample mean. Find the probability of the type I error, and the probability of the type II error when the true mean μ is 1.
   (b) What should the rejection region be in order to maintain the test level α at 0.05.
   (c) Will $n = 45$ be a sufficiently large sample to test the hypothesis at the $\alpha = 0.05$ level of significance if the experimenter wants the type II error probability to be no greater than 0.20 when $\mu = 2$?

2. A drapery store manager was interested in determining whether a new employee can install vertical blinds faster than an employee who has been with the company for two years. The manager takes independent samples of 10 vertical blind installations of each of the two employees and computes the following information. Assume the installation times are normally

	New Employee	Veteran Employee
Sample size	10	10
Sample Mean	22.2min	24.8min
Standard Deviation	0.90min	0.75min

distributed for both employees.
   (a) Is it reasonable to assume equality of variances in this problem? Set up an appropriate hypothesis and test it using the critical value approach.
   (b) Set up the hypothesis to test whether the new employee installs vertical blinds faster, on average, than the veteran employee and test the hypothesis using the p-value approach.

3. Suppose that $Y_1, \cdots, Y_n$ denote a random sample from a population having an exponential distribution with mean θ.
   (a) Derive the most powerful test for $H_0 : \theta = \theta_0$ against $H_a : \theta = \theta_a$ where $\theta_a < \theta_0$.
   (b) Is the test derived in the previous part uniformly most powerful for testing : $\theta = \theta_0$ against $H_a : \theta < \theta_0$

Table 1: The regression data

x	30	25	90	60	50	35	75	110	45	80
y	0.3	0.2	5.0	3.0	2.0	0.5	4.0	6.0	1.5	4.0

Table 2: The regression equation is $y = -1.74 + 0.0731x$

	Estimate	Standard deviation	T-ratio	p-value
β_0	-1.7359	0.1882	-9.22	0.000
β_1	0.073099	0.002867	25.50	0.000

4. A scientist is studying the relationship between x = inches of annual rainfall and y=inches of shoreline erosion. One study reported data as shown in Table 1. Table 2 contains part of the regression analysis results by MINITAB, a statistical software.

   (a) Write out the formula that is used to compute the estimate of β_1, and the one that is used to compute the standard deviation of β_1.

   (b) Based on the numbers provided in Table 2, construct a 95% CI for β_1.

   (c) The T-ratio for β_1 is the T test statistic for the hypothesis: $H_0 : \beta_1 = 0$ vs $H_1 : \beta_1 \neq 0$. Write out the formula that is used to compute the T-ratio.

   (d) For the same hypothesis as in (c), the p-value is used to determine whether to reject H_0. Explain how the p-value for β_1 is obtained and draw your conclusion for the hypothesis based on the p-value.

   (e) Let σ^2 be the variance of the random error in the simple linear regression model. The estimate of σ based on the above data is $S = 0.2416$. Using the fact that $(n-2)S^2/\sigma^2 \sim \chi^2_{n-2}$, perform an appropriate test for the hypothesis $H_0 : \sigma^2 = 1$ vs $H_1 : \sigma^2 \neq 1$. State your conclusion.