STAT 516 Spring 2017 Midterm

Name_____**ID**_____

–SHOW YOUR WORK WHERE WORK IS REQUIRED. NO WORK, NO CREDIT!
–IF YOU DON'T KNOW HOW TO DO A PROBLEM, MOVE ON AND COME BACK TO IT
–READ THE QUESTIONS CAREFULLY!! –GOOD LUCK

1. Let $Y_1, \cdots, Y_n$ be a random sample from a population with density function

$$f(y|\theta) = \begin{cases} \frac{3y^2}{\theta^3}, & 0 \le y \le \theta \\ 0, & \text{elsewhere} \end{cases}$$

Consider estimating θ with $\hat{\theta} = 4\bar{Y}/3$.

   (a) (9pts) Show that $\hat{\theta}$ is an unbiased estimator of θ.

   (b) (9pts) Find the standard error and estimation error bound of $\hat{\theta}$.

   (c) (9pts) With the estimator $\hat{\theta}$, find the sample size necessary to estimate θ to be within 0.05 with probability 0.95, if θ is approximately 1.

   (d) (9pts) When n is large, what is the approximate distribution of $\hat{\theta}$?

   (e) (9pts) Construct a 95% confidence interval based on the approximate distribution of $\hat{\theta}$ from the previous part.

   (f) (9pts) Is $\hat{\theta}$ MVUE? Elaborate.

2. Let $Y_1, \cdots, Y_n$ be a random sample from the Gamma(n, θ) distribution whose density function is given by

$$f(y|\theta) = \frac{1}{(n-1)!\theta^n} y^{n-1} e^{-y/\theta}$$

for $y > 0$. Note that $EY_1 = n\theta$ and $Var(Y_1) = n\theta^2$.

   (a) (9pts) Write out the likelihood of the sample.

   (b) (9pts) Find the MLE $\hat{\theta}$ of θ.

   (c) (8pts) Find the MLE for the variance of Y_1, which is $n\theta^2$.

3. In order to study Massachusetts voters' opinions about same sex marriages, two independent random samples are taken, one from Hamphsire and one from Franklin county. The following results are obtained:

Group	Sample size	Total in favor
Franklin	100	50
Hampshire	200	120

(a) (10pts) Give an estimate, an estimated standard error for that estimate and an approximate 90% confidence interval for the true proportion in favor in Franklin county. Be sure to identify each of the three items.

(b) (10pts) Using the same data from the previous part, give an estimate, an estimated standard error for that estimate and an approximate 95% confidence interval for the difference between the proportion in favor in Franklin and Hampshire college (defining the difference as Franklin - Hampshire). Be sure to identify each of the three items.