

STAT 516 Spring 2016 Midterm

Name _____ ID _____

–SHOW YOUR WORK WHERE WORK IS REQUIRED. NO WORK, NO CREDIT!
–IF YOU DON'T KNOW HOW TO DO A PROBLEM, MOVE ON AND COME BACK TO IT
–READ THE QUESTIONS CAREFULLY!! –GOOD LUCK

1. Let Y_1, \dots, Y_n be a random sample from the uniform distribution on the interval $(\theta, \theta + 1)$. Let

$$\hat{\theta}_1 = \bar{Y} - \frac{1}{2}$$

- (a) Show that $\hat{\theta}_1$ is an unbiased estimator of θ .
 - (b) Find the standard error and estimation error bound of $\hat{\theta}_1$.
 - (c) With the estimator $\hat{\theta}_1$, find the sample size necessary to estimate θ to be within 0.05 with probability 0.95, if θ is thought to be approximately 1.
 - (d) When n is large, what is the approximate distribution of $\hat{\theta}_1$?
 - (e) Construct a 95% confidence interval based on the approximate distribution of $\hat{\theta}_1$ from the previous part.
2. Let Y_1, \dots, Y_n be a random sample from the probability density function

$$f(y|\theta) = (\theta + 1)y^\theta$$

for $1 > y > 0$ and $\theta > -1$. Note that $EY_1 = (\theta + 1)/(\theta + 2)$ and $V(Y_1) = (\theta + 1)/(\theta + 3)$

- (a) Write out the likelihood of the sample.
 - (b) Find the MLE $\hat{\theta}$ of θ .
 - (c) Find the moment estimator for θ .
 - (d) Find the MLE for the variance of Y_1 .
3. For the following, you don't have to carry out the final calculation, but set up the answers with **all numerical values**.
- (a) A random sample of 25 vitamin tablets are collected and the amount of vitamin C (in mg) in each tablet is measured. A sample mean of 253 and a standard deviation of 9 mg are observed for the vitamin C content. Assume that the vitamin C content from a randomly chosen tablet follows a normal distribution with mean μ and variance σ^2 . Find a 90% confidence interval for the true mean vitamin C content μ .
 - (b) Using the same data and assumption from the previous part, find a 90% confidence interval for the true variance σ^2 .