STAT516 Exam I Spring 2006

Name:
Student ID:

1. (13 points) Suppose $X \sim N(\mu, \sigma^2)$ and $Y \sim N(\nu, \omega^2)$, and that X and Y are independent. Using moment generating functions to find the distribution of $U = 5X + 2Y$.

   Hint: You may assume that if $X \sim N(\mu, \sigma^2)$, then the moment generating function of X is given by
   $$M_X(t) = \exp\left[\mu t + \sigma^2 t^2/2\right].$$

2. (13 points) Suppose that $X_1, \cdots, X_n \overset{iid}{\sim} f_X(x;\theta)$ where
   $$f_X(x;\theta) = \frac{1}{2\theta}\exp(-|x|/\theta) \qquad (\theta > 0)$$
   Find the maximum likelihood estimator of θ.

3. (24 points) Suppose that $X_1, \cdots, X_n \overset{iid}{\sim} Uniform(\theta, \theta+1)$, where θ is a fixed but unknown constant that we wish to estimate.

   (a) (6 points) Show that the moment estimator of θ is $\bar{X} - 1/2$.

   (b) (6 points) Show that the moment estimator is unbiased.

   (c) (6 points) Write down the sampling distribution of the estimator when n is large.

   (d) (6 points) Construct a 95% confidence interval for θ based on the result in (c).