Math 441, Solutions to Midterm Spring 2005

1. "Closes out the position" means ABC takes short position on August 1 for 4 contracts.

   So (negative) profit on futures position is $4(8.00 - 9.00) = -\$4.00$. Real price paid is $0.75 * 50 = \$37.50$. So effective price is $37.50 + 4 = \$41.50$.

2. (a) Futures price is $250 * 1115 * e^{0.05*(10/12)} = 290610$ As always, initial value is \$0

   (b) Value after 6 months is
   $$-(S_{\frac{6}{12}} - 290610e^{-0.05(10/12-6/12)}) = -1200*250 + 290610e^{-0.05(10/12-6/12)} = -14190$$

   where $S_{\frac{6}{12}}$ is the spot price at six months of 250 shares of S&P. Note the negative sign is because you take a short position.

3. (a) We think of portfolio as one bond, with coupon in two years and one in 3 years. The usual duration formal gives $D = 2.65$ years The price of this portfolio (in denominator of formula for duration is about 2.5 million

   (b) She should short
   $$\frac{2.65 * 2.5 * 10^6}{2.5 * 985} = 2680$$
   of Bond AAA's.

   (c) Note that 5 basis points is 0.0005.
   $$\Delta B = -BD\Delta y = -2.5 * 10^6 * 2.65 * (0.0005) = \$ - 3300.$$

4. (a) Let r be 6 month and R be 12 month spot rate, then
   $$98 = 100e^{-r*0.5} \rightarrow r = 0.0404$$
   $$51 = 2e^{-r*0.5} + (2 + 50)e^{-R*1} \rightarrow R = 0.0586$$

   (b) We are trying so find x where
   $$(1 + x/4)^4 = e^{0.0404}$$
   This is $x = 0.0406$. note it should be more than $r = 0.0404$

   (c) A formula for the forward rate is given by
   $$\frac{R * 1 - r * \frac{1}{2}}{1 - \frac{1}{2}} = \frac{0.586 - 0.0404 * .5}{.5} = 0.768.$$

   (d) The yield y satisfies
   $$51 = 2e^{-y*0.5} + (2 + 50)e^{-y*1} \rightarrow y = 0.058$$

5. If F_0 were huge, like a zillion bucks, then Lex would: take a short position locking in F_0, borrow 1000, buy one ounce of krypto for the \$1000. In a year, he'd deliver the krypto for F_0 and pay back the loan of $1000e^{0.05*1} = 1051$ for a risk free profit of $F_0 - 1051$.

So $F_0 \leq 1051$.

If F_0 were tiny like one buck, then Lex would: take a long position locking in F_0, sell one ounce of his krypto for \$990. In a year, he'd getback the krypto for F_0 and withdraw the investment of $900e^{0.05*1} = 1041$ for a risk free profit of $1041 - F_0$.

So $1041 \leq F_0 \leq 1051$.