

Math 441, Solutions to Midterm Fall 2005

1. Futures price is

$$F = 16(400 + 5)(1 + 0.006/4)^2 = 6676.$$

2. Hedge ratio is

$$h = \rho \frac{\sigma_S}{\sigma_F} = (.08) * (0.25)/0.20 = 1.$$

On Oct 27, take long position in $1 * 1000 = 1000$ futures with delivery Dec 15. On Nov 27 take short position in 1000 futures. On Dec 15, exchange goods receive (pay) net gain (loss).

3. (a)

$$\begin{aligned} SD(\Delta X) &= \sqrt{E(\Delta X^2) - E(\Delta X)^2} \\ &= \sqrt{(0.45 * 10^2 + 0.55 * (-5)^2) - (0.45 * 10 + 0.55 * (-5))^2} \\ &= \$7.60 \end{aligned}$$

- (b) Expected profit from SHORT position is $(0.45) * (75 - 80) + (0.55) * (75 - 65) = 3.25$

4. (a) Let r_1 and r_2 be the one-year and two-year spot rates.

$$97 = 100 \exp\{-r - 1\}$$

implies $r_1 = 3.0\%$.

$$100 = 4 \exp\{-0.030\} + 104 \exp\{-2r_2\}$$

implies $r_2 = 3.9\%$. The forward rate is $(2 * 0.039 - 1 * 0.030)/(2 - 1) = 4.8\%$.

- (b) Let y be the yield. It solves

$$(2 * 97 + 100) = (2 * 100 + 4)e^{-y} + 104e^{-2y}$$

ie, $y = 3.5\%$, Duration and convexity are

$$\begin{aligned} D &= \frac{204e^{-0.035} + 2 * 104e^{-0.035*2}}{297} = 1.316 \\ C &= \frac{204e^{-0.035} + 4 * 104e^{-0.035*2}}{297} = 1.99. \end{aligned}$$

Let $\Delta y = 0.025$ be yield change, let B, B' be old and new price resp.

$$B' \approx B + (-DB\Delta y) + \frac{1}{2}CB(\Delta y)^2 = \$284.50.$$

5. The total gain is $(5.4 - 4.4) - (7.0 - 6.8) = 80$ basis points. 30 for (intermediary Japanese) bank, 25 each for A and B. B borrows US from outside paying 4.4 US. So it must receive 4.4 US from bank. It therefore pays bank $6.9 - 0.025 = 6.55$ yen. Bank wants gain in yen only so pays A 6.25 yen and receives 4.4 US. A borrows externally 7.0 yen. Check: A pays 7.0 - 6.25 yen and 4.4 US which is $5.4 - 0.025$.

Physically, A pays $(0.75)/2 * 1,200,000,000 = 4,500,000$ yen.

6. \$1500 is obviously too high. So today, Lex takes short position, borrows \$1000, buys 1 pound krypto. In a year, he delivers krypto and pays back loan for profit of $1500 - 1000(1 + 0.06/2)^2 = 439.10$. In today's dollars, that's $439.1(1 + 0.06/2)^{-2} = \413.89 .