Math 441 **Mathematics of Finance** **Fall 2013**

Midterm

October 17, 2013

Name: ____________________________

Instructions: Show all your work for full credit, and box your answers when appropriate. Unless otherwise noted, the risk free rate is per annum with continuous compounding and stocks do not pay dividends.

1. The risk-free rate is 5%. A one-year 600-strike European call on Google (GOOG) prices for 50. A one-year 700-strike European call on GOOG prices for 25. Consider a bear spread made of these two calls. What is the most profit (in one year from now) this spread can possibly have?

   Solution: Short the 600-strike, long the 700-strike. Receive $50 - 25 = 25$ at $t = 0$. Payoff is $-\max(S_1 - 600, 0) + \max(S_1 - 700, 0)$ which has a max of 0 (see graph). So max profit at $t = 1$ is $0 + 25e^{0.05} = 26.28$

   Note: watch out for time value of money!

2. The risk-free rate is 5%. The spot price of Microsoft (MSFT) is \$26. The stock will pay one dividend of \$1 in 3 months.

   - A 6-month European call on MSFT with strike 25 prices for \$5.00.
   - A 6-month European put on MSFT with strike 25 prices for \$1.50.
   - A 6-month European call on MSFT with strike 30 prices for \$1.50.

   Describe an arbitrage opportunity and compute (the present value of) its profit.

   Solution: Put-call parity is violated with the $K = 25$ strike options:

   $$c + Ke^{-rT} = 29.38 > 26.51 = p + S_0 - D$$

   where present value of dividend is $D = 1e^{-0.5\times .25}$. Do long put and stock, short call and bond. In 3 months deposit dividend. In 6 months close out position. Profit today is $29.38 - 26.51 = 2.87$.

   Note: there is no put-call parity relation between the call and put with *different* strikes!

3. The risk-free rate is 5%. The price of MSFT in 6 months has an equal probability of being one of four prices: \$20, \$25, \$30, \$35. (It can be no other price.) The spot price of MSFT is \$30.

Portfolio A is made of the following: long one at-the-money 6 month European call on MSFT; long one bond which pays \$100 in 6 months; short 3 MSFT shares (stocks). What is the probability that portfolio A's payoff is less than or equal to \$20?

Solution: If $S_{0.5}$ denotes the price of MSFT in 6 months, the payoff of Portfolio A is $\max(S_1 - 30, 0) + 100 - 3S_{0.5}$. This has a 25% chance of being $5 + 100 - 3 * 35 = 0, 0 + 100 - 3 * 30 = 10, 0 + 100 - 3 * 25 = 25, 0 + 100 - 3 * 20 = 40$. So a 50% chance of being \$20 or less. Note that convention for "payoff" is at $t = 0.5$, but even if you computed the present value of the payoff, it still has 50% chance of being less than \$20.

4. Facebook (FB) currently trades for \$50. In 3 month FB will increase by 12% or decrease by 8%. The risk-free rate is 3%. Portfolio X is a long position in seven at-the-money 3-month American calls on FB, expiring in 3 months. Use the binomial tree model to compute the price of Portfolio X today.

   Solution: We have $u = 1.12, d = 0.92$ so compute RNW probability $p = (e^{rT} - d)/(u - d) = 0.4376$. Since call is at-the-money, strike $K = 50$. Since X is 7 calls, up- and down-paryoffs in 3 months are $H_u = 7\max(S_u - K, 0) = 7\max(56 - 50, 0) = 42$ and $H_d = 7\max(S_d - K, 0) = 7\max(46 - 50, 0) = 0$, respectively. So price H is max of exercising now $(7 \times (50 - 50) = 0)$ or waiting $e^{-rT}(pH_u + (1 - p)H_d) = 18.24$.

   Note: for the last step, could have said that since no dividends, Amer call and Euro call have same price (Chap 6) and thus just price Euro call. This skips having to say "exercising now gets $7 \times (50 - 50) = 0$"

5. The risk-free rate is 5%. The spot price of Apple (APPL) is 490. In 6 months, APPL will sell for $450e^{0.03+0.2Z}$ where $Z \sim N(0, 1)$ is a standard normal variable with mean 0 and standard deviation 1. Is the real world risk-free, risk-averse, or risk-prone? Justify your answer with a calculation.

   Solution: If μ is the real world annualized rate of return, then μ solves $S_0 e^{\mu T} = E_{RW}[S_T]$. So

$$490e^{\mu \times 6/12} = E[450e^{0.03+0.2Z}] = 450e^{0.03}E[e^{0.2Z}] = 450e^{0.03}e^{0.2^2/2}$$

   Solving for μ we get

$$\mu = \frac{12}{6} * (\ln(450/490) + 0.03 + 0.2^2/2) = -0.07 = -7\%$$

   Since the risk-free rate is $5\% > -7\%$, the world is risk-prone.

   Could also directly compare expectations

$$E_{RW}[S_{6/12}] = E[450e^{0.03+0.2Z}] = 473.72 < 502.4 = 490e^{0.05\times 0.5} = E_{RNW}[S_{6/12}]$$