Math 441, Problems from old exams and practise sheets

Here are some problems given out as exam and practise problems fromm previous classes. This is NOT a complete review of all the concepts you need to know for the exam.

1. The three-month American call option on a non-dividend paying stock with strike price $24 is currently selling for $2. The three-month European put option on the same non-dividend paying stock with strike price $24 is currently selling for $3. The current stock price is $20. The risk-free interest rate is 10%.

    (a) Describe an arbitrage opportunity involving one call and one put.

    (b) How much risk-free profit does one make with the strategy from part (a)?

2. A European put option is available on a stock, costing $3 and with a strike price of $42. A European call option is available on the same stock. It costs $2 and has a strike price of $45. Assuming the rate of interest is zero, find the range of values of the stock price which are possible without creating an arbitrage situation.

3. A put option on a stock with a strike price of $34 costs $5. A put option on the same stock but with a strike price of $30 costs $2. What is the range of values one can charge for a put on the stock with strike price $32 which avoids creating an arbitrage opportunity. Assume the rate of interest is zero.

4. An 8 month call option on a stock, with a strike price of $60 costs $2.40. An 8 month put option on the same stock, with a strike price of $50 costs $3.50. The risk-free rate of interest is 7% per annum, continuously compounded. Find the range of values of the stock price, 8 months from now, for which the strangle portfolio of the two options is in profit

5. A stock price is currently $35. The volatility of the stock is 20% per annum. The stock pays a continuous dividend of 2%. What is the (Black-Scholes) price of a nine month straddle with strike price $40? Assume the risk-free rate is 5%.

6. A non-dividend paying stock has volatility of 28% per annum and its current price is $54. The risk-free interest rate is 7% per annum, continuously compounded. Find the Black-Scholes price of a European put option on the stock with strike price of $55 and time to maturity of 4 months.

7. A stock pays a dividend (once) of one dollar in 12 months. The current stock price is $20. The risk-free interest rate is 10%. A European put option on this stock with strike price $24 and expiring in 1.5 years is currently selling for $2. What can you say about the price of the European

call, the American put and the American call on this stock, all with the same strike price and expiration date?

8. Consider a straddle on a stock with a strike price of $32. Describe how to construct the straddle with put(s) and/or call(s). Describe the pay-off (NOT the profit) function of the straddle, either with a formula or a well-drawn and (very) explicitly labeled graph.

   Suppose the straddle you constructed expires in 6 months. Suppose you know that the current stock price is $30 and over the next six months it is expected to rise to $33 or fall to $27. The risk-free interest rate is 6%. What should the straddle cost?

9. A stock price is currently $40. Over the next six months it is expected to go up by 10% or down by 10%. The risk-free rate is 5% per annum. What is the value of a six month American put option with a strike price of $38?

10. A bank pays six-month LIBOR and receives 8% per annum (with semi-annual compounding) on a swap with a notional principal of $100 million. (Cash payments are exchanged every six months.) The swap expires in 15 months. The LIBOR rates with continuous compounding for all maturities are 10%. The six-month LIBOR rate at the last payment date was 11% (with semi-annual compounding). What is the value of the swap to the bank?

11. The following questions are all multiple choice. You do not need to justisfy your answer. Assume no-arbitrage and/or the Black-Scholes formula when necessary. A European put has

    (a) A positive delta and positive gamma.
    (b) A positive delta and negative gamma.
    (c) A negative delta and positive gamma.
    (d) A negative delta and negative gamma.

12. The market price of a European call is $3.50 and its Black-Scholes price is $3.00. The market price of a put with the same strike and expiration date is $1.00. What does the Black-Scholes model say the price of the put should be?

13. Suppose a portfolio is worth $90 million and the S&P 500 trades at $1000. If the value of the portfolio mirrors perfectly the value of the index, how much of what type of options should be purchased to provide protection against the value of the portfolio falling below $70 million in 5 months time?

14. A bond with principal $1000 expires in 18 months and pays 8%-coupons semi-annually. The 6-month, 12-month and 18-month spot rates are all 5%. Calculate the duration of the bond.

15. Companies X and Y have been offered the following rates (in %) per annum on a $20 million 10-year loan.

Company	Fixed Rate	Floating Rate
X	8.2	LIBOR + 0.4
Y	7.2	LIBOR

Company X requires a fixed-rate loan; company Y requires a floating-rate loan. Design a swap that will net a bank, acting as an intermediary, 20 basis points, and that will appear equally attractive to X and Y.

16. The current price of IBM stock is $60. Assume the **daily** volatility of IBM is 1%. The risk-free rate is 5% per annum. A bank is long 300 IBM stocks, and has written 50 European call options on IBM stock each with strike price $50 expiring in 9 months.

Calculate the delta and Gamma of the bank's portfolio using the Black-Scholes formula.

17. The current price of ATT stock is $40. Over the next two six-month periods the stock is expected to go up or down by 10%. The risk-free rate is 10%. Find the range of values for which it is never best to exercise after one six month period, an American put with a strike price in such a range.

18. Below are a number of different possible portfolios of options, all on the same underlying asset, S, whose current value is $30, and which does not pay dividends. Assume that the risk-free interest rate will remain constant at 5% for the next 6 months. For each portfolio, calculate δ and Γ.

- *Portfolio 1*: 1 3-month call option with strike price $30
- *Portfolio 2*: Portfolio 1 plus and one share of stock
- *Portfolio 3*: Portfolio 2, plus and one 10-year zero coupon bond with face value $100
- *Portfolio 4*: Portfolio 3 plus one option which pays $(S - K)^2$
- *Portfolio 5*: Portfolio 4 plus one 6-month futures contract for the purchase of a share of S, at delivery price $32.

19. Suppose that a portfolio is worth $12,000,000$, and that it has $\beta = 1.5$ with respect to a certain stock index, whose current value is $900. Further suppose that the current risk-free interest rate is 8% per annum, and that the dividend yields of the portfolio and the index are 3% and 4% per annum, respectively. If the value of the index in 6 months turns out to be $950, what is the value of the portfolio? What if the value of the index in 3 months is $1000, $850, or $800.

20. Using the Black-Scholes formula, determine the price of a 3-month call option with strike price 40 on a stock whose current price is $38. The risk-free interest rate is 4%, and the stock price volatility is 15% per annum. Now consider that the this stock has a carrying cost, which is continuously calculated as 1%. How does this affect the Black-Scholes prediction of the price.

Using put-call parity for options with costs-of carry, determine the price of the put option corresponding to the call option