Math 441 **Mathematics of Finance** **Spring 2006**

Final
May 19, 2006

Name: _____

Instructions: Show all your work for full credit, and box your answers when appropriate. There are 7 questions, each worth 10 points. Unless otherwise noted, all risk free rates are per annum with continuous compounding.

1. The contract size of a futures in oil is 1000 barrels. The futures price of one of these contracts with delivery date in 6 months is \$80,000. Suppose it costs \$4 (to paid up front) to store each barrel of oil for 6 months. The risk-free rate is 5%. Compute the spot price of one barrel of oil.

2. The spot price of biodiesel is \$70 a barrel. It costs \$2 (to paid in 6 months) to store one barrel of biodiesel for 6 months. The risk-free rate is 7%. A European call on one barrel with strike price \$68 and expiring in 6 months costs \$10. A European put on one barrel with strike price \$68 and expiring in 6 months costs \$3. Describe an arbitrage opportunity involving one call and one put and one barrel. List what cash flows occur when. What is the risk-free profit of this opportunity?

3. Consider Bond A which is a 6%-coupon bearing bond maturing in 15 months, with principal \$100. Coupons are paid semi-annually. Suppose all spot rates are 5% with continuous compounding. This problem continues on the next page.

   (a) Compute the duration of the Bond A.

   (b) Suppose the Fed raises rates by 25 basis points. Use the duration to estimate the new price of the Bond A.

(c) Bond B is priced at \$88 and has a duration of 2 years. Suppose your portfolio is long two of Bond A. PRIOR to the Fed raising any rates (so spot rates are still 5%), you decide to hedge against yield shifts by trading Bond B. What do you do?

4. The spot price of Google is \$360. The price will increase or decrease by 11% every 6 months for the next year. The risk free rate is 5%. Use a two-step binomial tree to price an AMERICAN put on Google, with strike price \$380 and expiring in 1 year.

5. Let S_t denote the the price of Google stock at time t. Today is $t = 0$. Assume $S_0 = 360$.. The price will increase or decrease by 11% in the next six months. The risk free rate is 5%. Define the function $g(x)$ by

$$\begin{aligned} g(x) &= 10 \text{ if } 0 \leq x \leq 350 \\ g(x) &= 10 + 2x \text{ if } 350 \leq x \leq 400 \\ g(x) &= 110 \text{ if } x \geq 400 \end{aligned}$$

(a) Show how to construct the derivative whose payoff is $g(S_{6/12})$ using Google puts, Google calls and/or bonds.

(b) Use a one-step binomial tree to price this derivative.

6. Let S_t denote the the price of Google stock at time t. Today is $t = 0$. Assume $S_0 = 360$. Assume the log-normal model for the stock price. The risk free rate is 5%. The expected return is 10%. The annual volatility is 20%. This problem continues on the next page.

   (a) Use the Black-Scholes model to price a Bear Spread on Google, involving one call which is at-the-money, and another call which is in-the-money by \$40.

(b) Without using the Black-Scholes formula explicitly (ie, do not type more numbers into the calculator), but just using your knowledge of the Greeks, what can you say (if anything) about the price of the bear spread compared to your answer in (a) if the spot price of Google were \$361 compared to \$360? Justify your answer.

7. Let S_t denote the the price of Google stock at time t. Today is $t = 0$. Assume $S_0 = 360$. Assume the log-normal model for the stock price. The risk free rate is 5%. The expected return is 10%. The annual volatility is 20%. Let $X = \ln(S_{7/12}^3)$. This problem continues on the next page.

   (a) Find a 70% confidence interval for for X.

   (b) Compute today's price of a derivative which pays you $\$X$ in 7 months.

(c) Compute the standard deviation of X.