

**Midterm**  
May , 2005

Name: \_\_\_\_\_

**Instructions:** Show all your work for full credit, and indicate your answers clearly.  
There are eight (8) questions.

1. Companies A and B have been offered the following rates per annum on a \$20 million 10-year loan.

Company	Fixed Rate	Floating Rate
A	8.2	LIBOR + 0.4
B	7.2	LIBOR + 0.1

Company A requires a fixed-rate loan; company B requires a floating-rate loan. Design a swap that will net a bank, acting as an intermediary, 30 basis points, and that will appear equally attractive to A and B.

2. The current asset price is \$40. The risk free rate is 5% (unless otherwise noted, all rates are continuously compounded). It costs, today, \$2 to store the asset for 6 months. The asset is expected to pay a dividend of \$5 in 3 months. A futures contract has a contract size of 10 assets. Compute the delivery price for one contract in 6 months.

3. The current price of 5CollegeInc is \$50 per share. It has a volatility of 20% and an expected return of 10%. The risk-free rate is 8%. The stock does not pay dividends. It can be shown that the futures price for one share, to be delivered in 1 year, is approximately \$54.16. Assume such futures contracts have contract size one share. This problem continues on the next page.
- (a) Compute the Black-Scholes price of a European call option on this stock, with strike price \$48 and expiring in 9 months.

- (b) The Darth is a derivative made up of the following financial instruments:
- two call options as described in part(a);
  - one put with the same specifications as the call.
  - 1000 nickels to be handed over in 6 months;
  - a long position in one futures contract with delivery date in 1 year and delivery price of \$54.16;
  - and 2 shares of the stock, to be handed over in 15 months.

Compute the price of the Darth.

4. The stock price today, at  $t = 0$  is  $S_0 = \$30$ . The stock price in 6 months, at  $t = 1/2$ , is expected to go up to \$34 or down to \$27. That is  $S_{1/2}$  will be 34 or 27. The buick is a derivative whose payoff is  $\$|S_t - 26 - 2t|$  where  $|x|$  is the absolute value of  $x$ . But it is an American-style derivative, as the owner can decide when (at what  $t$ ) to receive the pay-off. Assume the risk free rate is 5%. Compute today's price of the buick.

5. Let  $S_t$  be the price of a stock at time  $t$  with  $t = 0$  being today. Assume  $S_0 = \$80$ . The volatility of the stock is 30% per annum. The expected return of the stock is 12%. The risk-free rate is 10%. Let  $f(S_1) = 0$  if  $S_1 \leq 70$ ,  $f(S_1) = S_1 - 70$  if  $70 \leq S_1 \leq 75$  and  $f(S_1) = 5$  if  $S_1 \geq 75$ . This problem continues on the next page.
- (a) Graph  $y = f(S_1)$ . What is the strategy (derivative) called whose pay-off is given by this graph?

- (b) How do you construct this derivative? Be specific.

(c) Use Black-Scholes to price this derivative?

(d) Assuming Black-Scholes, compute the delta of this derivative.

6. Let  $S_t$  be the price of a stock at time  $t$  with  $t = 0$  being today. Assume  $S_0 = \$70$ . The volatility of the stock is 20% per annum. The expected return of the stock is 12%. The risk-free rate is 10%. This problem continues on the next page.

(a) What is the expected stock price in 3 months?

(b) What is the standard deviation of the stock price in 3 months?



(c) Find an 86% confidence interval for the stock price in 3 months.

(d) What is the daily volatility of the stock?

7. A fund manager has a portfolio worth \$1 million. The beta is 1.0. The current S&P 500 spot price is \$1000. The volatility of S&P is 20%. The risk-free rate is 8%. The S&P does not expect to pay any dividends in the next year. One S&P futures contract has a contract size of 250 shares. The fund manager is wary of market movements over the next 6 months.

(a) If the manager wants to hedge against market movements over the next 6 months using S&P futures, what exactly should she do?

(b) If the manager wants to avoid losing more than \$100,000 at the end of 6 months, and she wants to use options on the S&P, what exactly should she do?

8. Consider a stock, whose spot price is  $S_0$  and which will pay a dividend between now and time  $T$  whose present value is  $\$D$ . Let  $C$  be the price of an American call expiring at time  $T$  for this stock with strike  $K$ . Let  $P$  be the price of an American put expiring at time  $T$  for this stock with strike  $K$ . Use a no-arbitrage argument to prove that  $P + S_0 - D \leq C + K$ .