Final

December 20, 2005

Name: ______________________________

Instructions: Show all your work for full credit, and indicate your answers clearly.
There are seven (7) questions.

1. Portfolio X is made up of two bonds: Bond A is a 2-year zero-coupon bond with principal \$100, Bond B is a 1-year 10%-coupon bond (coupons paid semi-annually) with principal \$80. The zero-rates are the following: 6 month rate is 5%, 12 month rate is 5.1%, 18 month rate is 5.2%, 24 month rate is 5.3%. Unless otherwise noted, all rates are per annum and continuously compounded.

   Compute the price and yield of the Portfolio X.

   If the Fed lowers all interest rates, would the price of Portfolio X go up or down? You do not need to estimate the change in price.

2. The current exchange rate is 1.20 US dollars for 1 Euro. Companies A and B have been offered the following rates per annum on a 12 million US Dollar loan and 10 million Euro loan. Interest payments are to be made every six months.

Company	US Dollars	Euros
A	8.2	8.2
B	7.2	7.7

Company A wishes to borrow US Dollars and company B wishes to borrow Euros. Design a swap that will net a bank, acting as an intermediary, 30 basis points, and that will appear equally attractive to A and B. Be sure that the bank assumes all the risk associated to exchange rates.

What is the net amount of Euros and US Dollars that the bank pays/receives at each exchange of coupons?

3. The current asset price is \$40. The risk free rate is 5%. The asset price has a volatility of 20% and an expected return of 10%. It costs \$2 (to be paid up front) to store the asset for 6 months. The asset is expected to pay a dividend of \$5 in 3 months. This problem continues on the next page.

   (a) Compute the Black-Scholes price of a European call option on this asset, with strike price \$38 and expiring in 6 months.

(b) **Without** using the Black-Scholes formula, deduce from your answer in (a) what the (Black-Scholes) price of the 6-month European put option with strike \$38 should be. You may confirm your answer using the Black-Scholes formula, but you will not receive any credit for such a method.

4. The stock price today, at $t = 0$ is $S_0 = \$40$. The stock price in 8 months, at $t = 8/12$, is expected to go up to \$46 or down to \$36. That is $S_{8/12}$ will be 46 or 36. Assume the risk free rate is 5%. This problem continues on the next page

   (a) Describe how to construct a butterfly spread with strikes \$30, \$40, \$50 and then compute today's price of the butterfly.

(b) Given that the real world is risk-averse instead of risk-neutral, what, if anything, can you say about the (real world) probability that the stock will be \$46 in eight months from now. (One sentence at most.)

5. Let S_t be the price of a stock at time t with $t = 0$ being today. Assume $S_0 = \$80$. The volatility of the stock is 30% per annum. The expected return of the stock is 12%. The risk-free rate is 10%. Let $g(x) = \ln(x^3) + \frac{17}{x}$. The ProfessorX derivative gives you in 7 months: $\$g(S_{7/12})$ and a one-hundred dollar bill. This problem continues on the next page.

   (a) Compute the price of ProfessorX.

(b) Compute the Gamma of ProfessorX.

6. Consider the humpback spread which is

   - long a call with strike \$50,
   - short a call with strike \$55,
   - short a call with strike \$65, and
   - long a call with strike \$70.

   All calls are European and expire in 8 months. The risk-free rate is 0%. Let $c_{50}, c_{55}, c_{65}, c_{70}$ denote the price of these four calls today. This problem continues on the next page.

   (a) Graph the payoff (not profit) of the humpback spread.

(b) Next to each pair of quantities, list which one is bigger (or more precisely, which one is greater than or equal to the other). Assume only No-Arbitrage. If not enough information is provided to determine which one is bigger, write down "inconclusive." No justification needed.

i. c_{50} or c_{55}

ii. c_{50} or $c_{55} + c_{65}$

iii. $c_{50} + c_{70}$ or $c_{55} + c_{65}$

iv. c_{65} or $c_{50} + c_{70}$

v. c_{50} or 50

7. Let S_t be the price of Cisco at time t with $t = 0$ being today. Assume $S_0 = \$75$. The volatility of the stock is 30% per annum. The expected return of the stock is 12%. The risk-free rate is 10%. This problem continues on the next page. If you cannot solve part (a), pretend the answer is 0.25 and proceed to part (b).

   (a) What is the probability, in the Risk-Neutral World, that the stock price in 9 months from now will be \$75 or less?

(b) A Digital Option is an "all-or-nothing" type of derivative. Consider the Digital Cisco Option which will pay you (in 9 months) \$12 if Cisco is trading at or below \$75 in 9 months, and will pay you nothing if Cisco is trading above \$75 in 9 months. Compute the price of the Digital Cisco Option.

Hint: Let P be a probability for the sample space Ω. Let $A \subset \Omega$ be an event. Let k be a constant. If X is a random variable such that

- $X(\omega) = k$ for any outcome ω in A
- $X(\omega) = 0$ for any outcome ω not in A,

then $E[X] = kP(A)$.