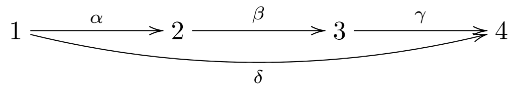
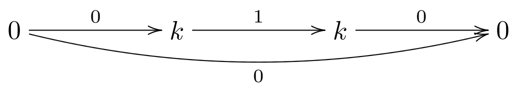
AARMS Summer School 2008
Representation Theory - Third Assignment

1. Let Q be the quiver

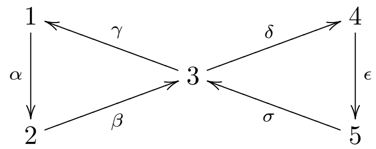


   and let M be the indecomposable representation



   Compute $L_1 = \tau M, L_2 = \tau^2 M$ and $L_3 = \tau^3 M$ using the Nakayama functor. Find three representations N_1, N_2 and N_3, by explicitly writing out the matrices, such that $\underline{\dim} N_i = (1,1,1,1)$ and L_i is a subrepresentation of N_i, for $i = 1, 2, 3$.

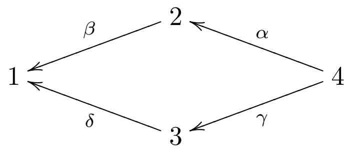
2. Compute the Auslander-Reiten quiver of (Q, I), where Q is the quiver



   and $I = \{\alpha\beta, \beta\gamma, \gamma\alpha, \delta\epsilon, \epsilon\sigma, \sigma\delta\}$. [Hint: use a triangulated polygon]

3. Let e be a non-trivial *central* idempotent (central means $ea = ae, \forall a \in A$). Show that $A \cong eA \oplus (1-e)A$ *as algebras*. [The direct sum of two algebras A, B is the vector space $A \oplus B$ with multiplication $(a, b)(a', b') = (aa', bb')$]

4. Let Q be the quiver



   and let $I_1 = <\alpha\beta + \gamma\delta>$ and $I_2 = <\alpha\beta - \gamma\delta>$ be two admissible ideals of kQ. Show that

   (a) $I_1 \neq I_2$ if $\operatorname{char} k \neq 2$.

   (b) there exists an isomorphism of algebras

$$kQ/I_1 \xrightarrow{\cong} kQ/I_2$$