

AARMS Summer School 2008
Representation Theory - Second Assignment

1. Let Q be a quiver without oriented cycles, let $M, N \in \text{rep } Q$ and let $g : M \rightarrow N$ be a surjective morphism in $\text{rep } Q$. Furthermore, let $i \in Q_0$ and let $P(i)$ be the indecomposable representation at i of Definition 24. Show that for every morphism $f : P(i) \rightarrow N$ there exists a morphism $h : P(i) \rightarrow M$ such that $f = g \circ h$.
2. Let Q be the quiver

$$\begin{array}{ccccccc}
 1 & \xrightarrow{\alpha} & 2 & \xrightarrow{\beta} & 3 & \xrightarrow{\gamma} & 4 \\
 & & & & & & \nearrow \\
 & & & & & & \delta \\
 & & & & & & \searrow
 \end{array}$$

List all indecomposable projective representations of Q and compute a projective resolution

$$0 \rightarrow P_1 \rightarrow P_0 \rightarrow S(1) \rightarrow 0.$$

[Only compute the representations P_0, P_1 , do not specify the maps.]

3. Let $D = \text{Hom}_k(-, k)$ and $(-)^t = \text{Hom}(-, A)$ be the dualities of section 6.1 and 6.2. Show that $\forall i \in Q_0$
 - (a) $D(P_Q(i)) = I_{Q^{op}}(i)$,
 - (b) $(P_Q(i))^t = P_{Q^{op}}(i)$.
4. Compute the Auslander-Reiten quiver of

$$Q = 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \leftarrow 5 \leftarrow 6$$