Math 235 Quiz Septembe 30, 2013

Given a matrix

$$A = \begin{pmatrix} 1 & 2 & -1 & 3 & 4 \\ -1 & -2 & 0 & 1 & -1 \\ 0 & 0 & -1 & 4 & 3 \end{pmatrix}$$

we have that the kernel of A is spanned by the three vectors

$$\begin{pmatrix} -2 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 4 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 3 \\ 0 \\ 1 \end{pmatrix}.$$

The RREF of A is

$$\begin{pmatrix} 1 & 2 & 0 & -1 & 1 \\ 0 & 0 & 1 & -4 & -3 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

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The problem is to find vectors so that they span the image and there are no redundancies.

- First we get a set of vectors that span the image. Note that A gives a function $A : \mathbb{R}^5 \to \mathbb{R}^3$., so the image is a subspace of $\mathbb{R}^3$.
- Let $e_i = (0 \cdots 1 \cdots 0)$ be the vector of length 5 with all o's except a 1 in the i-th position. Let A_i be the i column of A.Then

  $$Ae_i = A_i.$$

  From this we conclude that the column vectors of A span the image of A.
- The basis of the kernel that we have computed gives us relations among the column vectors. In particular it gives us that the column vectors corresponding to the free variables are redundant. For example using the second element in our basis of the kernel we have

  $$A \begin{pmatrix} 1 \\ 0 \\ 4 \\ 1 \\ 0 \end{pmatrix} = 1A_1 + 4A_3 + 1A_4 = 0 \quad \text{We have used an element of the kernel.}$$

  This implies that $A_4 = -A_1 - 3A_3$.

We conclude that a basis of the image is the set of column vvectors of A corresponding to the leading ones: $\{ \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \end{pmatrix} \}$.