

## M534H HOMEWORKS– Spring 2009

• **Set 9. Due date: Thursday May 7**

Section 5.3 1, 2a)b), 3, 4a)b), 5a), 6, 10.

Section 5.4 1, 5, 6, 7, 8a)

• **Set 8. Due date: Thursday April 30**

Section 4.3 2, 11

Section 5.1 3, 5, 6, 7, 9.

Section 5.2 3, 4, 7, 9, 10, 11, 17.

• **SET 7. Due date: Thursday April 16th**

Section 4.1: 2, 3

Section 4.2: 1, 3

Additional Problems Separate variables to solve the following problems.

A1.  $u_{tt} - u_{xx} = 0$  in  $0 < x < 3$  with boundary conditions  $u(0, t) = u(3, t) = 0$

A2.  $u_t = u_{xx}$  in  $0 < x < L$  with boundary conditions  $u_x(0, t) = u_x(L, t) = 0$

A3.  $u_t = u_{xx} + u$  in  $0 < x < \pi$  with boundary conditions  $u(0, t) = u(\pi, t) = 0$

• **SET 6. Due date: Thursday April 9th**

Section 3.1: 1, 2

Section 3.2: 5

Section 3.4: 1, 3

• **SET 5. Due date: Thursday March 26th–Delayed. Due at Exam April 3rd**

Section 2.4: 1, 2, 7 (you can use that the integral is  $\sqrt{\pi}$  as proved in class)  
8, 14, 15 16, 18

Section 2.5: 1

• **SET 4. Due date: Thursday March 12th**

Section 2.3: 2, 4, 5, 6, 7.

• **SET 3. Due date: Thursday March 5th**

Section 2.1: 1, 2, 5, 8, 9, 11

**Problem 11.** Find the general solution of

$$3u_{tt} + 10u_{xt} + 3u_{xx} = \sin(x + t)$$

Section 2.2: 1, 2, 3, 5\*

**Problem 5\*** Consider the wave equation in 1D with *damping*

$$u_{tt} = c^2 u_{xx} - ku - ru_t \quad k, r > 0$$

show that the *energy functional*

$$E(t) = \frac{1}{2} \int_{-\infty}^{\infty} |u_t|^2 + c^2 |u_x|^2 + k|u|^2 dx$$

satisfies  $dE/dt \leq 0$ ; that is *energy decreases*. Assume  $u$  and its derivatives vanish as  $x \rightarrow \pm\infty$ .

**Additional Problems.**

**Problem A.1.** Find the solution to wave equation with initial conditions  $u(x, 0) = \sin x$ ,  $u_t(x, 0) = 0$ . Calculate then  $u_t(0, t)$ .

**Problem A.2.** Solve the PDE  $u_{xx} + u_{xt} - 10u_{tt} = 0$

• **SET 2. Due date: Thursday February 26th**

Section 1.4: 1

Section 1.5: 1, 4 and then do 5, 6 as modified below.

Problem 5 states: Consider the equation

$$u_x + yu_y = 0$$

with boundary condition  $u(x, 0) = \phi(x)$ .

(a) For  $\phi(x) = x$ , for all  $x$ , show that no solution exists.

(b) For  $\phi(x) = 1$  for all  $x$ , show that there are many solutions.

Problem 6 states: Solve the equation  $u_x + 2xy^2 u_y = 0$  and find a solution that satisfies the auxiliary condition  $u(0, y) = y$ .

Section 1.6: 1, 2, 4

Additional Problems:

(1) Find the general solution of  $u_x - \sin(x) u_y = 0$ . Then find the special solution that satisfies  $f(0, e^y) = e^y$ .

(2) Find the regions in 2d-space where  $x^2 u_{xx} + 4u_{xy} + y^2 u_{yy} = 0$  is respectively elliptic, parabolic, hyperbolic. Plot these regions.

• **SET 1. Due date: Thursday February 12th**

Section 1.1: 2, 3, 4, 11

Section 1.2: 1, 2, 3, 4, 5.

Section 1.3: 7, 9, 10 (in  $\mathbb{R}^3$ ), 11.

Additional Problem:

Let  $u = u(\mathbf{x}, t)$  be a solution to the wave equation  $u_{tt} - \Delta u = 0$  in  $\mathbb{R}^2$ . Assuming that  $\nabla u \rightarrow 0$  fast enough as  $|\mathbf{x}| \rightarrow \infty$  prove that

$$E(t) = \int_{\mathbb{R}^2} |u_t|^2 + |\nabla u|^2 \, dx dy$$

is constant in  $t$  for all time  $t$ .