1. [12%] For the Trapezoidal Rule with $n = 4$, we have $\Delta x = \dfrac{2-0}{4} = 1/2$. Then:

$$\begin{aligned}\int_0^2 f(x)\,dx \approx T_4 &= \frac{\Delta x}{2}\left[f(0) + 2f\left(\frac{1}{2}\right) + 2f(1) + 2f\left(\frac{3}{2}\right) + f(2)\right]\\ &= \frac{1/2}{2}\left(1 + 2\cdot\frac{4}{5} + 2\cdot(-1) + 2\cdot 0 + 1\right)\\ &= \frac{1}{4}\cdot\frac{8}{5} = \boxed{\frac{2}{5}}\end{aligned}$$

Or you could set this up using a table:

i	x_i	$f(x_i)$	w_i	$w_i f(x_i)$
0	0	1	1	1
1	1/2	4/5	2	8/5
2	1	-1	2	-2
3	3/2	0	2	0
4	2	1	1	1
			$\sum w_i f(x_i)$	8/5

And then:

$$\begin{aligned}\int_0^2 f(x)\,dx \approx T_4 &= \frac{\Delta x}{2}\sum_{i=0}^{4} w_i f(x_i)\\ &= \frac{1/2}{2}\cdot\frac{8}{5} = \frac{2}{5}\end{aligned}$$

2. [12%] $\displaystyle\int \left(x\sqrt{x} + 5e^{-5x}\right)\,dx = \int \left(x^{3/2} + 5e^{-5x}\right)\,dx = \boxed{\frac{2}{5}x^{5/2} - e^{-5x} + C}$

3. [12%] Begin by using parts:

$$u = \arctan x \qquad dv = dx$$
$$du = \frac{1}{1+x^2}\,dx \qquad v = x$$

$$\begin{aligned}\int \arctan x)\,dx &=\\ &= x\arctan x - \int \frac{x}{1+x^2}\,dx\end{aligned}$$

In the last integral explicitly (or implicitly) substitute:

$$\begin{aligned}u &= 1 + x^2\\ du &= 2x\,dx\\ \frac{1}{2}\,du &= x\,dx\end{aligned}$$

$$\begin{aligned}\int \frac{x}{1+x^2}\,dx &= \frac{1}{2}\int\frac{1}{u}\,du\\ &= \frac{1}{2}\ln(|u|) + C\\ &= \frac{1}{2}\ln(|1+x^2|) + C\\ &= \frac{1}{2}\ln|1+x^2| + C\end{aligned}$$

(The absolute value may be removed inside ln because $1 + x^2 > 0$.) Then:

$$\int \frac{x}{1+x^2}\,dx = \boxed{x\arctan x - \frac{1}{2}\ln|1+x^2| + C}$$

4. **[12%]** Factor $x^2 - 5x + 4 = (x-1)(x-4)$. Use partial fractions:

$$\begin{aligned}
\frac{3x}{(x-1)(x-4)} &= \frac{A}{x-1} + \frac{B}{x-4} \\
3x &= A(x-4) + B(x-1) \\
x = 1: \quad 3 &= -3A \implies A = -1 \\
x = 4: \; 12 &= \quad 3B \implies B = \quad 4
\end{aligned}$$

$$\begin{aligned}
\int \frac{3x}{x^2 - 5x + 4}\, dx &= \int \left(\frac{-1}{x-1} + \frac{4}{x-4} \right) dx \\
&= \boxed{-\ln|x-1| + 4\ln|x-4| + C} \quad \text{or } = \ln\left|\frac{(x-4)^4}{x-1}\right| + C
\end{aligned}$$

5. **[12%]** *Method 1:* keep in terms of sin and cos.

$$\int \frac{\sin^3 x}{\cos^5 x}\, dx = \int \frac{\sin^2 x}{\cos^5 x} \cdot \sin x\, dx = \int \frac{1 - \cos^2 x}{\cos^5 x} \cdot \sin x\, dx$$

Substitute:

$$\begin{aligned}
u &= \cos x \\
du &= -\sin x\, dx \qquad\qquad -du = \sin x\, dx
\end{aligned}$$

$$\begin{aligned}
\int \frac{\sin^3 x}{\cos^5 x}\, dx &= -\int \frac{1-u^2}{u^5}\, du = -\int \left(\frac{1}{u^5} - \frac{1}{u^3} \right) du = -\int \left(u^{-5} - u^{-3}\right) du \\
&= -\left(-\frac{1}{4}u^{-4} + \frac{1}{2}u^{-2} \right) + C = \frac{1}{4}u^{-4} - \frac{1}{2}u^{-2} + C = \frac{1}{4u^4} - \frac{1}{2u^2} + C \\
&= \boxed{\frac{1}{4\cos^4 x} - \frac{1}{2\cos^2 x} + C}
\end{aligned}$$

Note: It would be confusing to write $\cos^{-4} x$ and $\cos^{-2} x$.

Method 2: convert to tan and sec.

$$\int \frac{\sin^3 x}{\cos^5 x}\, dx = \int \frac{\sin^3 x}{\cos^3 x} \cdot \frac{1}{\cos^2 x}\, dx = \int \tan^3 x \sec^2 x\, dx$$

Substitute:

$$\begin{aligned}
u &= \tan x \\
du &= \sec^2 x\, dx
\end{aligned}$$

$$\begin{aligned}
\int \tan^3 x \sec^2 x\, dx &= \int u^3\, du \\
&= \frac{1}{4}u^4 + C \\
&= \frac{1}{4}\tan^4 x + C
\end{aligned}$$

6. **[12%]** Start with:

$$\int \frac{x^5}{(x^3+1)^2}\,dx = \int \frac{x^3}{(x^3+1)^2}\,x^2\,dx$$

Substitute:

$$u = x^3 + 1 \implies x^3 = u - 1$$

$$du = 3x^2\,dx \implies \frac{1}{3}\,du = x^2\,dx$$

$$\begin{aligned}
\int \frac{x^5}{(x^3+1)^2}\,dx &= \frac{1}{3}\int \frac{u-1}{u^2}\,du = \frac{1}{3}\int \left(\frac{1}{u} - \frac{1}{u^2}\right) du \\
&= \frac{1}{3}(\ln|u| + \frac{1}{u}) + C \\
&= \boxed{\frac{1}{3}\left(\ln\left|x^3+1\right| + \frac{1}{x^3+1}\right) + C}
\end{aligned}$$

7. **[16%]** *Method 1:* First find the indefinite integral.

Substitute:

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$\begin{aligned}
\int \frac{1}{x(\ln x)^{3/2}}\,dx &= \int \frac{1}{u^{3/2}}\,du = \int u^{-3/2}\,du \\
&= -2u^{-1/2} + C = -\frac{2}{\sqrt{u}} + C \\
&= -\frac{2}{\sqrt{\ln x}} + C
\end{aligned}$$

Then:

$$\begin{aligned}
\int_e^{\infty} \frac{1}{x(\ln x)^{3/2}}\,dx &= \lim_{B\to\infty} \int_e^{B} \frac{1}{x(\ln x)^{3/2}}\,dx \\
&= \lim_{B\to\infty} \left(-\frac{2}{\sqrt{\ln x}}\right)\Bigg|_e^B = \lim_{B\to\infty}\left(-\frac{2}{\sqrt{\ln B}} + \frac{2}{\sqrt{\ln e}}\right) \\
&= \lim_{B\to\infty}\left(-\frac{2}{\sqrt{\ln B}} + \frac{2}{\sqrt{1}}\right) \\
&= 0 + 2 = \boxed{2}
\end{aligned}$$

Method 2: Work directly with definite integral.

Substitute:

$$u = \ln x$$

$$du = \frac{1}{x}\,dx$$

$$x = B \implies u = \ln B$$

$$x = e \implies u = \ln e = 1$$

$$\begin{aligned}
\int_e^{\infty} \frac{1}{x(\ln x)^{3/2}}\,dx &= \lim_{B\to\infty} \int_e^{B} \frac{1}{x(\ln x)^{3/2}}\,dx \\
&= \lim_{B\to\infty} \int_1^{\ln B} \frac{1}{u^{3/2}}\,du = \lim_{B\to\infty} \int_1^{\ln B} u^{-3/2}\,du \\
&= \lim_{B\to\infty} \left(-2u^{-1/2}\right)\Big|_1^{\ln B} = \lim_{B\to\infty}\left(-\frac{2}{\sqrt{u}}\right)\Bigg|_1^{\ln B} \\
&= \lim_{B\to\infty}\left(-\frac{2}{\sqrt{\ln B}} + \frac{2}{\sqrt{1}}\right) = 0 + 2 = 2
\end{aligned}$$

8. **[12%]** Begin by completing the square:

$$2x - x^2 = -\left(x^2 - 2x\right) = -\left(x^2 - 2x + 1\right) + 1 = 1 - (x-1)^2$$

Substitute:

$$u = x - 1,$$
$$du = dx.$$

$$\int \sqrt{2x - x^2}\ dx = \int \sqrt{1-(x-1)^2}\ dx = \int \sqrt{1-u^2}\ du$$

Use trig substitution there:

$$u = \sin\theta$$
$$du = \cos\theta\, d\theta$$
$$\sqrt{1-u^2} = \sqrt{1-\sin^2\theta} = \sqrt{\cos^2\theta} = \cos\theta$$

$$= \int \cos\theta\cos\theta\ d\theta = \int \cos^2\theta\ d\theta$$
$$= \int \frac{1}{2}\left(1+\cos 2\theta\right)\ d\theta$$
$$= \frac{1}{2}\left(\theta + \frac{1}{2}\sin 2\theta\right) + C$$
$$= \frac{1}{2}\left(\theta + \frac{1}{2}2\ \sin\theta\cos\theta\right) + C = \frac{1}{2}\left(\theta + \sin\theta\cos\theta\right) + C$$

From

$$\sin\theta = u, \qquad \cos\theta = \sqrt{1-u^2},$$

$$\int \sqrt{2x-x^2}\ dx = \frac{1}{2}\left(\arcsin u + u\sqrt{1-u^2}\right) + C$$
$$= \boxed{\frac{1}{2}\left[\arcsin(x-1) + (x-1)\sqrt{1-(x-1)^2}\right] + C}$$
$$= \frac{1}{2}\left[\arcsin(x-1) + (x-1)\sqrt{2x-x^2}\right] + C$$