Algebra 411.1

Homework 4

Due Thursday October 13, in class.

All answers should be justified!

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Problems from the book. 3.6, 3.7, 4.13,

1. (a) Show that the subset $A = \{0, 2, 4\}$ of $(\mathbb{Z}_6, +_6)$ is a subgroup.

(b) Determine the order of the group $(\mathbb{Z}_n, +_n)$.

(c) Determine the order of each element of $(\mathbb{Z}_8, +_8)$.

2. Recall that the set $GL_2(\mathbb{R})$ of $2{\times}2$ matrices with a nonzero determinant (i.e., the invertible matrices), is a group for the operation of matrix multiplication. Show that the following subsets of $GL_2(\mathbb{R})$ are subgroups :

(1) The *diagonal matrices*

$$H \stackrel{\text{def}}{=} \{\begin{pmatrix} a & 0 \\ 0 & c \end{pmatrix} \in GL_2(\mathbb{R})\}.$$

(2) The upper triangular matrices

$$B \stackrel{\text{def}}{=} \{\begin{pmatrix} a & b \\ 0 & c \end{pmatrix} \in GL_2(\mathbb{R})\}.$$

3. We say that two elements a, b in a group $(G, \cdot)$ *commute* if $a{\cdot}b = b{\cdot}a$. The center $Z(G)$ of a group G is defined as the subset of all $a \in G$ that commute with *all* elements of G.

(a) Prove that the center $Z(G)$ is a subgroup of G.

(b) The group $(G, \cdot)$ is said to be *commutative* or *abelian*) if for any two elements a, b of G we have $a{\cdot}b = b{\cdot}a$. If G is commutative what is the center $Z(G)$ of G?

(c) Prove that cyclic groups are commutative.[1]

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[1] A group G is *cyclic* if it contains an element g which *generates* G in the sense that G consists of all powers of g.

0. Read the notes on the web page: chapters 0-4.