Algebra 412

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Homework 1

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Due Wednesday Feb 11, in class.

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1. Prove that $(\mathbb{Z}_n, +_n, \cdot_n)$ is a ring.

2. If $(R, +_R, \cdot_R)$ and $(S, +_S, \cdot_S)$ are rings prove that $R \times S$ with operations

$$(r, s) + (r', s') \stackrel{\text{def}}{=} (r +_R r', s +_S s') \quad \text{and} \quad (r, s)\cdot(r', s') \stackrel{\text{def}}{=} (r \cdot_R r', s \cdot_S s')$$

is also a ring.

3. If $(R, +, \cdot)$ is a ring prove that $M_n(R)$, the set of $n \times n$ matrices with values in R, is also a ring where operations on matrices are defined as usual:

$$(A + B)_{ij} \ = \ A_{ij} + B_{ij} \quad \text{and} \quad (A \cdot B)_{ij} \ = \ \sum_{k=1}^{n} A_{ik} \cdot B_{kj}.$$

4. Show that

$$\mathbb{Z}[i, \sqrt{2}] \ = \ \{a + bi + c\sqrt{2} + di\sqrt{2};\ a, b, c, d \in \mathbb{Z}\}$$

is a subring of the ring $\mathbb{C}$ of complex numbers.

5. Show that the function

$$f : \ \mathbb{Z}_6 \longrightarrow \mathbb{Z}_2 \times \mathbb{Z}_3, \quad f(k) \stackrel{\text{def}}{=} k(1, 1) \ \text{ for } \quad k = 0, 1, 2, 3, 4, 5;$$

is an isomorphism of rings.