University of Massachusetts
Department of Mathematics and Statistics
Math 235, Linear Algebra
Spring 2014, Course Syllabus

Course Description:

Basic concepts of linear algebra. Matrices, determinants, systems of linear equations, vector spaces, linear transformations, and eigenvalues. Prerequisite or corequisite: MATH 132, or consent of instructor. (Gen.Ed. R2)

[Note: Because this course presupposes knowledge of basic math skills, it will satisfy the R1 requirement upon successful completion.]

Text:

- Otto Bretscher, *Linear Algebra with applications,* Fifth Edition, Pearson Education 2012, or
- Otto Bretscher, *Linear Algebra with applications,* Fourth Edition, Pearson Prentice Hall 2009.

Course Goals: The course goals are divided into content/process learning outcomes.

Content Learning Outcomes:

Upon completion of this course, the successful student will be able to:

1. Use Gaussian elimination to do all of the following: solve a linear system with reduced row echelon form, solve a linear system with row echelon form and backward substitution, find the inverse of a given matrix, and find the determinant of a given matrix.
2. Demonstrate proficiency at matrix algebra. For matrix multiplication demonstrate understanding of the associative law, the reverse order law for inverses and transposes, and the failure of the commutative law and the cancellation law.
3. Use Cramer's rule to solve a linear system.
4. Use cofactors to find the inverse of a given matrix and the determinant of a given matrix.
5. Determine whether a set with a given notion of addition and scalar multiplication is a vector space.
6. Determine whether a given subset of a vector space is a subspace.
7. Determine whether a given set of vectors is linearly independent, spans, or is a basis.
8. Determine the dimension of a given vector space or of a given subspace.
9. Find bases for the null space, row space, and column space of a given matrix, and determine its rank.
10. Demonstrate understanding of the Rank-Nullity Theorem and its applications.
11. Given a description of a linear transformation, find its matrix representation relative to given bases.
12. Demonstrate understanding of the relationship between similarity and change of basis.
13. Find the norm of a vector and the angle between two vectors in an inner product space.
14. Use the inner product to express a vector in an inner product space as a linear combination of an orthogonal set of vectors.
15. Find the orthogonal complement of a given subspace.
16. Demonstrate understanding of the relationship of the row space, column space, and nullspace of a matrix (and its transpose) via orthogonal complements.
17. Demonstrate understanding of the Cauchy-Schwartz inequality and its applications.
18. Determine whether a vector space with a given form is an inner product space.
19. Use the Gram-Schmidt process to find an orthonormal basis of an inner product space. Be capable of doing this both in $\mathbf{R}^n$ and in function spaces that are inner product spaces.
20. Use least squares to fit a line ($y = ax + b$) to a table of data, plot the line and data points, and explain the meaning of least squares in terms of orthogonal projection.
21. Use the idea of least squares to find orthogonal projections onto subspaces and for polynomial curve fitting.
22. Find (real and complex) eigenvalues and eigenvectors of 2×2 or 3×3 matrices.
23. Determine whether a given matrix is diagonalizable. If so, find a matrix that diagonalizes it via similarity.
24. Demonstrate understanding of the relationship between eigenvalues of a square matrix and its determinant, its trace, and its invertibility/singularity.
25. Correctly define terms and give examples relating to the above concepts.
26. Be familiar with basic theorems about the above concepts.
27. Prove or disprove statements relating to the above concepts.
28. Be adept at hand computation for row reduction, matrix inversion and similar problems.

Process Learning Outcomes:

- Students will be able to understand linear algebra problems from three perspectives of analysis, algebra, and geometry.
- Students will be able to appropriately apply linear algebra methods.
- Students will improve their technical reading and writing skills in the context of linear algebra.

General Education:

The purpose of the General Education requirement is to stretch students' minds, broaden their experiences, and prepare them for 1) Their college experiences and subsequent professional training; 2) Their careers and productive lives; 3) Community engagement and informed citizenship; 4) A diverse and rapidly changing world; and 5) A lifetime of learning.

Math 235 carries the General Education designation of R2: *Quantitative Reasoning*. Besides the overall goals of the General Education program (see http://www.umass.edu/gened), this course meets the following objectives specific to the Quantitative Reasoning requirement.

1. Content: Know fundamental questions, ideas, and methods of inquiry/analysis used in the discipline.

Many problems occurring in applications within the physical sciences, engineering, social sciences, and beyond, can be translated into solving a number m of linear equations in a certain number n of variables. The central theme of linear algebra is the study of solutions of such equations. Besides the applications, the theoretical foundation developed in this course for understanding solution sets of linear equations form a foundation for all higher algebra. Students will be trained in understanding linear algebra concepts from multiple viewpoints: analytical, algebraic, and geometric.

2. Critical Thinking: Students demonstrate capacity for making comparisons and developing critical acuity.

Students in this course engage in a particularly pure form of Critical Thinking involved in mathematical sciences, namely those involving logical and numerical relationships. These notions are infused in the material throughout the entire course. Although the primary emphasis is on on problem-solving and understanding concepts, students will also be introduced to the rigorous language of higher mathematics and its proof techniques.

3. Communication: Demonstrate capacity to express one's thoughts in writing – regular practice in writing encourages clear thinking and clear expression.

Much of the grade in this course is based on performance on two midterm exams and a final exam. Some questions from these exams and homework assignments require clear communication of ideas in a written form. Partial credit is given -- it is not the case that we simply seek a numerical answer, but the instructors analyze, with great care and patience, what the student is able to express in terms of the sequence and logical flow of ideas, particularly in word problems.

4. Demonstrate capacity to apply disciplinary perspectives and methods of analysis to real world problems (the larger society) or other contexts.

The course has two simultaneous components which share air time throughout the course. Namely, the course introduces abstract notions such as vector spaces, linear transformations, eigenvalues, etc. and relates these to the measurement of quantities with direct applications in the sciences and engineering. Students learn how to take a concrete problem such as "How does one analyze the current flowing through a complicated electrical circuit?" and create a set of mathematical ideas modeling the problem. They then apply techniques of linear algebra to solve the mathematical problem and relate its solution to answer the real-world problem.

Calculator: Calculators will **not** be allowed in the exams. You may use calculators to check your homework solutions, but credit will be given only for answers showing all your steps (unless mentioned otherwise in the assignment).

Common Exams (50%): There will be two common exams: a midterm and a final. These will be given and graded in common. Each exam is worth 25% of the course grade.

Common midterm: Wednesday, April 2, from 7:00 to 9:00 PM.

Final Exam: To be scheduled during the exam period. The last day of this period is Thursday, May 8. **Make-ups will not be given to accommodate travel plans.**

Instructor's Grade (50%): Each instructor will determine 50% of the student's grade based on the student's performance in the class (e.g., in the section's first exam, quizzes, homeworks, class attendance and participation).

The score on each of the three parts (Common midterm, Final Exam, and Instructor's Grade) will be scaled to the 0-100 scale. The average of these grades will determine the course letter grade.

Course letter-grade scale

A	A-	B+	B	B-	C+	C	C-	D+	D	F
87	83	78	75	72	68	62	59	55	50	<50

Weekly schedule & Course Topics

Week of	Sections covered	Events
1/21	1.1, 1.2, start 1.3	Monday is winter break, MWF sections to catch up in week 5
1/27	1.3, 2.1, start 2.2	
2/3	2.2, 2.3	
2/10	2.4, 3.1	
2/17	3.2, 3.3	Mon: President day, Tues: Mon. Sched.
2/24	3.3, 3.4	
3/3	4.1, 4.2	Last day to drop with W: 3/6
3/10	4.3 (in part), 6.1	
3/17	SPRING BREAK	
3/24	6.2, 6.3 (lightly), start 7.1	
3/31	7.1, 7.2	Common midterm: Wednesday, 4/2, 7:00 to 9:00 PM
4/7	7.3, 7.4	
4/14	7.5, 5.1	
4/21	5.2, 5.3 (start)	Mon: Patiots day, Wed: Mon schedule
4/28	5.3	Last day of classes: Wednesday, April 30
5/5	Final examinations resume	Last day of final exams: Thursday, May 8

1.1. Introduction to Linear Systems
1.2. Matrices, Vectors, and Gauss-Jordan Elimination
1.3. On the Solutions of Linear Systems; Matrix Algebra,
2.1. Introduction to Linear Transformations and Their Inverses
2.2. Linear Transformations in Geometry
2.3. Matrix Products
2.4. The Inverse of a Linear Transformation
3.1. Image and Kernel of a Linear Transformation
3.2. Subspaces of R^n; Bases and Linear Independence
3.3. The Dimension of a Subspace of R^n
3.4. Coordinates
4.1. Introduction to Linear Spaces
4.2. Linear Transformations and Isomorphisms
4.3. The Matrix of a Linear Transformation
6.1. Introduction to Determinants
6.2. Properties of the Determinant
6.3. Geometrical Interpretations of the Determinant (skip Cramer's Rule)
7.1. Dynamical Systems and Eigenvectors: An Introductory Example
7.2. Finding the Eigenvalues of a Matrix

7.3. Finding the Eigenvectors of a Matrix
7.4. Diagonalization
7.5. Complex Eigenvalues
5.1. Orthogonal Projections and Orthonormal Bases
5.2. Gram-Schmidt Process and QR Factorization
5.3. Orthogonal Transformations and Orthogonal Matrices