Homework 5

Exercise 1: The *standard deviation* $\sigma(X)$ of a random variable is the square root of the variance $\mathrm{Var}(X)$

$$\sigma(X) = \sqrt{\mathrm{Var}(X)}\,.$$

and it characterizes the "spread" of the random variable X. If a random variable X has expected value μ and standard deviation σ, then X takes values which are on average at distance σ from μ.

Consider the following application to investing. You can invest in a US stock fund and its rate of return X, that is the return on every invested dollar is a random variable with mean .15 and standard deviation $\sigma(X)$. You can also invest in an Asian real estate fund, its rate of return Y is a random variable with mean .15 and standard deviation $\sigma(Y)$. In this case the standard deviation is a *measure of the risk*, since the larger the standard deviation, the more likely it is to have rate of return which is far from the mean. This can be both good or bad, since the rate of return could be either much larger than the average or much smaller.

It is well known that to make your investment safer you should diversify. Assume that you decide invest a proportion α of your fortune in the US stock fund and a proportion $1-\alpha$ in the Asian real estate fund and *assume* that the rate of returns are independent. Determine the optimal proportion α^* which will minimize your risk.

Exercise 2: I hand you a coin and make the claim that it is biased and that heads comes up with probability .48% of the times. You decide to flip the coin yourself a number of times to make sure that I was saying the truth. Use Chebyshev inequality to estimate how many times you should flip that coin to be sure that the coin is biased with a probability of .95.

Exercise 3:

1. Consider the uniform random variable U on $1, 2, \cdots, N$, with probability distribution $P(U = n) = \frac{1}{N}$ for $n = 1, 2 \cdots, N$. Compute the mean and the variance of U. *Hint:* You may use the equalities

   $$1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

   and

   $$1^2 + 2^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}\,.$$

   (Do you know how to prove these?)

2. Billy sells newspaper at the entrance of the metro. Every morning he buys 200 papers for \$1 each and sells them for \$1.50. He goes home after all his papers are sold or at noon. He can return unsold newspapers to the distributor for \$ 0.5 a piece. The demand for his newspaper is uniform between 151 and 250 and if the demand exceeds 200 then the demand is unfilled. Compute the expected value and the standard deviation of Billy's earning.

Exercise 4: Three musicians pick seven pieces of music out of 22 pieces independently of each other. They play one piece of music together only if the piece is chosen by all three of them. Let X be the random variable which denote the number of pieces of music played. Determine the expected value and the variance of X and compute $P\{X \geq 1\}$.
Hint: To do this let X_i to be 1 if the three musicians picks the i^{th} music piece and 0 otherwise. Express X in terms of X_i. Are the X_i's independent?