Homework 4

Exercise 1:

1. At a certain casino card game which has paying odds of 1 to 1, your probability of winning each game is $p = 0.494$. You walk into the casino with \$25 dollars with the goal to get \$500. Compute the probability for you to succeed if you use the following strategies

   (a) You make repeated \$5 bets until you either win \$500 or you are wiped out.

   (b) You make repeated \$25 bets until you either win \$500 or you are wiped out.

   (c) You play bold strategy.

2. By an extremely clever card counting trick you managed to change the odds of that game to $p = .502$. Compute again the probability that you succeed if you use the following strategies

   (a) You make repeated \$5 bets until you either win \$500 or you are wiped out.

   (b) You make repeated \$25 bets until you either win \$500 or you are wiped out.

   (c) You play bold strategy.

Exercise 2: Compute the bold strategy probability, $Q(1/7)$, $Q(2/7)$, $Q(3/7)$, $Q(4/7)$, $Q(5/7)$, and $Q(6/7)$.

Exercise 3: If you go to the casino with \$100 in your pocket and decide to play roulette with the goal to increase your fortune to a total of \$900. Since you have been sitting in a class on gambling and other silly stuff at U of M, you decide to use the bold strategy. But then you think: "well, what if i would use bold strategy on other bets than red/black?"

1. Give a recursive formula for the probability $Q(z)$ to reach a fortune 1 starting with a fortune z $(0 < z < 1)$ if you bet on a subset of 12 numbers out of 36, (the payout is \$2 if you bet \$1 and win, that is your fortune is multiplied by 3 if you win.)

2. Decide what is the best of strategy for your fortune to reach \$900 starting with \$100 at roulette: use bold strategy on red/black or use bold strategy by betting on a subset of 12 numbers?

3. What do you think would be even better?

Exercise 4: Suppose you play a series of fair games ($p = q$) with bets of fixed size (say 1).

1. If you start with a fortune of j, what is the probability that you play forever?
2. If you have an infinite amount of money, what is the probability that you ever achieve a gain of M units.

Exercise 5: The probability to win at powerball have been computed in a previous exercise (you can find them at `http://www.powerball.com/powerball/pb_prizes.asp`). Remember that a ticket is worse \$2. The expected amount paid back by the lottery for a bet depends on the jackpot. Today (February 24, 2015) the jackpot is \$70 million with a cash value of \$45.8 million. Compute your expected gain if you take the cash value.

Exercise 6: Poker dice is a carnival game played with 5 dice with 9, 10, J, Q, K, A on the sides instead of the usual numbers. The bettor chooses two different faces (let us say he chooses Q and K) from the six choices and is paid at rate of 1:1 if both sides appear (that is they each appear at least once on the 5 dice). Compute your expected gain at this game. *Hint:* Compute the probability not to win and use the formula for $P(A \cup B)$.

Exercise 7: An *life annuity* is the promise by A (typically a life insurance company or a pension fund) to pay B (typically an individual) a certain sum of money for the rest of his life. For example in many pensions systems, when you retire, you are promised a fixed amount of money every year for the rest of your life, depending on how much you saved throughout your life (and your age as well).

To compute the value of an annuity usually one uses

$$\boxed{\text{Value of the annuity} = \text{Expected amount paid to B by A}}$$

1. Assume that A promises to pay B \$25,000 every year until B's death. To make things simple we say 1 unit = \$25,000. Based on public health data A estimates that the probability that B dies in any single year is $1 - p$. Assume that the first payment $X_0 = 1$ is made at once and we denote by X_j the amount paid at the end of year j (If B is alive at that time he gets a payment on one unit). Compute $E[X_j]$ and then the total value of the annuity.
2. The previous calculation ignores the fact that money can earn interest rate. Suppose that A can invest money with a return of α % per year (compounded yearly). If A needs to pay 1 unit to B in year i, how much money does A needs to have at the beginning of year 0. Based on this compute the value of the annuity, that is how much money needs A to have on hand at year 0 to pay the annuity in the future.

3. If you can invest money at a return of α % per year show that if you invest $1 + \frac{1}{\alpha}$ units you can obtain a income of 1 unit per year for ever. (Here too assume that the first payment X_0 is made at once).

Exercise 8: Consider the martingale betting systems where the probability of winning a single game is p and your limited fortune (or the house limit) allows to you bet at most k times.

1. What is the probability that you loose everything exactly on your $n^t h$ visit to the casino?

2. Compute the expected number of visits to the casino?
   Hint: How do you compute the series $\sum_{n=1}^{\infty} n x^{n-1} = 1 + 2x + 3x^2 + 4x^3 + \cdots$?