

Homework 3

Exercise 1: Solve the following difference equations

1. $2x(n) - 5x(n-1) = 0, x(0) = 2$
2. $2x(n) - 5x(n-1) = 3, x(0) = 3$
3. $2x(n) - 5x(n-1) = 3n, x(0) = 0.$
4. $2x(n+1) - 7x(n) + 3x(n-1) = 0, \quad x(0) = 1, x(1) = 2$
5. $2x(n+1) - 7x(n) + 3x(n-1) = 2 + 2^n, \quad x(0) = 3, x(1) = 0$

Exercise 2: Your mortgage is a 30 year fixed rate mortgage at a fixed annual rate of 4% compounded monthly.

1. If you borrow \$150'000 today, what is the total amount of money will you pay back to the bank during the next 30 years?
2. You decide that you can make a down-payment of \$15000 and that \$1250 is the maximal monthly payment you are willing to commit to. What is the value of the most expensive house can you buy?

Exercise 3: Your retirement account has a fixed rate of 8% per year *paid yearly*. You start saving for retirement at age 30 with a target retirement age of 65 and \$0 in your saving account. Set-up and solve a suitable first order difference equations to answer the following questions (compute all interests and payment on a yearly basis).

1. Suppose you set aside \$500 every month. How much money will you have for your retirement?
2. You want to retire with \$500,000. How much should you save every month?
3. Assuming that your salary is going to increase 5 % per year during your lifetime you also decide you contribution should follow suit and your monthly contribution increase by 5% every year. If your starting contribution is \$500 every month how much money will you have saved at retirement age?
4. Assuming again that your contribution is increasing by 5% every year, what should your starting contribution be if if you want to reach \$500,000 by retirement age?

Exercise 4: The Powerball jackpot for Feb 14, 2015 is \$ 40 millions. If you win it you are given the choice to either receive a lump sum of \$ 27 million or to receive \$ $1\frac{1}{3}$ million per year for the next 30 years. You go first to a financial advisor who tells you that you can invest your money with him and receive a guaranteed interest rate of α percent yearly (compounded annually). To compare your two options you think of invest all your money for the next 30 years and see how the two option compare.

1. Write down two difference equations for each of the two options (denote by $x(n)$ the value of investment after n years in million dollars).
2. For which interest rate α is the option of a lump sum better?