Homework 1

Exercise 1: As seen in class if there are 23 people in a room the probability of having two people with the same birthday is more than 1/2. In our class of 42 people what is this probability? For comparison compute the probability that somebody has the same birthday than you in our class? Are you surprised by the result?

Exercise 2: Bob and Maria are taking a math class with final grades A, B or C. The probability that Bob gets a B is .3 and the probability that Maria gets a B is .4. The probability that neither gets an A but at least one gets a B is .1. What is the probability that at least one gets a B but neither gets a C?

Exercise 3:

1. What odds should you give in favor of the following event?
   (a) A card chosen at random from a 52-card deck is an ace.
   (b) Exactly two heads will turn up when three coins are tossed.
2. In a soccer tournament race the odds that Arsenal wins are given 2 to 5 while the odds that Liverpool wins are 1 to 5. Give the odds that either Arsenal or Liverpool wins?

Exercise 4: (Betting odds vs odds) At the time you bet on horse races, the betting house will give you odds attached to each horse. They are called the betting odds. These odds are computed from the total amount of money bet on every horses up that time (the "pool") and the odds will determine your payout if you bet on that horse *and* the race happens right now. If more money comes in then the odds might change. So of the odds on Bernoulli are 11 to 2 it means that your bet of \$5 will pay you \$ 32.5. The betting odds are computed as follows. The betting house take a percentage of the money (%15 is about right) and redistribute all the rest to the winner. Assume that there are four horses and that the following amount of money has been betted on the horses.

Euler	525
Bernoulli	900
Laplace	225
Fourier	350

1. According to the amount bet by all the players compute the odds that each horse will win.
2. What are the betting odds given to you by the betting house (assuming they take a %15 cut).

Exercise 4: A six card hand is dealt from an ordinary deck of 52 cards. Find the probability that

1. All six cards are hearts
2. There are three aces, two kings and one queen.
3. There three cards of one suit and three of another suit.

Exercise 5: Compute the probabilities to obtain the following poker hands

1. Two pairs
2. A straight flush: fives cards of the same suit in order (e.g. 6, 7, 8, 9, 10 of hearts).
3. A flush : five cards of the same suit but not in order (e.g. 3, 5,6, queen,and king of spades).

Exercise 6: The powerball is a popular lottery organized by the multistage lottery association where 5 white balls are drawn out of a drum which 59 balls and one red ball is drawn out of another drum with 35 balls. The balls are drawn without replacement and the order in which balls are drawn does not matter. The prize of the ticket is $2 and there are 9 ways to win given in the table below. Compute the corresponding probabilities.

Balls	Prize	Probabilities
5 white & 1 red	Jackpot	
5 white	$ 1,000,000	
4 white & 1 red	$10,000	
4 white	$100	
3 white	$100	
3 white	$7	
2 white & 1red	$7	
1 white & 1red	$4	
1 red	$4	

See `http://www.powerball.com/powerball/pb_howtoplay.asp` for more details.

Exercise 7: Explain with a "story proof" why the identity

$$\binom{2n}{n} = \sum_{j=0}^{n} \binom{n}{j}^2$$

holds.
Hint: Think of a group of consisting of n boys and n girls.

Exercise 8: Prove the formula

$$P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)$$

by using the formula for two events. If you are courageous, prove by induction the formula for n events.