MATH 331.1, Fall 2007 : Quizz #6 solution

Your Name: ____

The quizz has 1 question worth 10 points.

Find the *general solution* for the second order system $\dfrac{d^2y}{dt^2} + 25y = \cos(t)$.

Solution: The homogeneous equation $\dfrac{d^2y}{dt^2} + 25y = 0$ is solved by trying an exponential e^{st}. Inserting into the equation gives

$$s^2 + 25 = 0\,, \quad s = \pm i\sqrt{5}\,.$$

The solution of the homogeneous equation is thus

$$y_h(t) = k_1 \cos(5t) + k_2 \sin(5t)\,.$$

In order to find a particular solution we use complex numbers and consider first the equation $\dfrac{d^2y}{dt^2} + 25y = e^{it}$. We look for a solution of the form $y_c(t) = ae^{it}$. Inserting into the equation gives

$$ai^2e^{it} + 25ae^{it} = e^{it}$$

that is $a(-1+25) = 1$ or $a = 1/24$. The particular complex solution of $\dfrac{d^2y}{dt^2} + 25y = e^{it}$ is

$$y_c(t) = \frac{1}{24}e^{it}\,.$$

Since $\cos(t)$ is the real part of e^{it} a particular solution $y_p(t)$ of $\dfrac{d^2y}{dt^2} + 25y = \cos(t)$ is obtained by taking the real part of $y_c(t) = \frac{1}{24}e^{it} = \frac{1}{24}(\cos(t) + i\sin(t))$, i.e.

$$y_p(t) = \frac{1}{24}\cos(t)\,.$$

The general solution $\dfrac{d^2y}{dt^2} + 25y = \cos(t)$ is then

$$y(t) = y_h(t) + y_p(t) = k_1 \cos(5t) + k_2 \sin(5t) + \frac{1}{24}\cos(t)\,.$$