# MATH 331.1, Fall 2007 : Quizz #3

**Your Name:** ________________________________________

The quizz has 1 question worth 10 points.

1. Consider the linear system $\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}$ where $A = \begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix}$.

(a) (5 points) Compute the eigenvalues and eigenvectors of the matrix $A$

To compute the eigenvalue we have

$$\begin{aligned} \det \begin{pmatrix} 1-\lambda & 3 \\ 1 & -1-\lambda \end{pmatrix} &= (1-\lambda)(-1-\lambda) - 3 \\ &= \lambda^2 - 4 \end{aligned}$$

Therefore the eigenvalues are $\lambda_1 = +2$ and $\lambda_2 = -2$.

For the eigenvalue $\lambda_1 = 2$ the eigenvector equation is

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 2 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

or

$$v_1 + 3v_2 = 2v_1 \qquad \text{and} \qquad v_1 - v_2 = 2v_2$$

Both equation gives $v_1 = 3v_2$ and so we can choose $v_1 = 3$ and $v_2 = 1$.

For the eigenvalue $\lambda_2 = -2$ the eigenvector equation is

$$\begin{pmatrix} 1 & 3 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = -2 \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$$

or

$$v_1 + 3v_2 = -2v_1 \qquad \text{and} \qquad v_1 - v_2 = -2v_2$$

Both equation gives $v_1 + v_2 = 0$ and so we can choose $v_1 = 1$ and $v_2 = -1$.

$$\lambda_1 = 2 \qquad \mathbf{V}_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix} \qquad\qquad \lambda_2 = -2 \qquad \mathbf{V}_2 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

(b) (4 points) Solve the initial value problem $\frac{d\mathbf{Y}}{dt} = A\mathbf{Y}$ with $\mathbf{Y}(0) = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$.

The general solution is given by

$$Y(t) = k_1 e^{2t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} + k_2 e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Setting $t = 0$ we find

$$\begin{pmatrix} 0 \\ 3 \end{pmatrix} = k_1 \begin{pmatrix} 3 \\ 1 \end{pmatrix} + k_2 \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

or

$$0 = 3k_1 + k_2 \qquad \text{and} \qquad 3 = k_1 - k_2$$

The first equation gives

$$k_2 = -3k_1$$

Inserting into the second equation gives

$$3 = k_1 + 3k_1$$

and thus we find

$$k_1 = \frac{3}{4} \qquad k_2 = -\frac{9}{4}$$

$$\mathbf{Y}(t) = \frac{3}{4}e^{2t} \begin{pmatrix} 3 \\ 1 \end{pmatrix} + \frac{9}{4}e^{-2t} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \frac{9}{4}e^{2t} + \frac{9}{4}e^{-2t} \\ \frac{3}{4}e^{2t} - \frac{9}{4}e^{-2t} \end{pmatrix}$$

(c) (1 points) Sketch the $x(t)$ and $y(t)$ graphs for the solution to the inital value problem in (b).