

## MATH 331.1, Fall 2007 : Quiz #2 Solution

Your Name: \_\_\_\_\_

The quiz has 2 questions worth 5 points each.

1. Solve the initial value problem  $\frac{dy}{dt} = 6y + e^t + 2e^{2t}$ ,  $y(0) = 3$ .

**Solution:** The general solution consists of the general solution of the homogeneous equation  $\frac{dy}{dt} = 6y$  plus a particular solution. The general solution of the homogeneous equation is

$$y(t) = Ke^{6t}.$$

In order to find a particular solution we try a function of the type

$$y(t) = ae^t + be^{2t},$$

where  $a$  and  $b$  are constants to be determined. Inserting this function into the equation leads to

$$ae^t + 2be^{2t} = 6ae^t + 6be^{2t} + e^t + 2e^{2t}.$$

Comparing the coefficients of  $e^t$  and  $e^{2t}$  gives the equations  $a = 6a+1$ , and  $2b = 6b+2$  with solution  $a = -1/5$  and  $b = -1/2$ .

The general solution is then

$$y(t) = Ke^{6t} - \frac{1}{5}e^t - \frac{1}{2}e^{2t}.$$

The initial condition  $y(0) = 3$  gives the equation  $3 = K - 1/5 - 1/2$  or  $K = 3.7$  so the solution of the initial value problem is

$$y(t) = 3.7e^{6t} - \frac{1}{5}e^t - \frac{1}{2}e^{2t}.$$

2. A couple of years ago, after careful analysis, a team of scientists came up with the following systems of differential equations describing the interaction between populations of frogs and alligators in a swamp. The alligators eat the frogs. The population of frogs is denoted by  $F(t)$  and the population of alligators is denoted by  $A(t)$ ; and  $t$  is measured in years.

$$\begin{aligned}\frac{dF}{dt} &= 6F - 10AF \\ \frac{dA}{dt} &= -2A + 1.2AF\end{aligned}$$

(a) The intensive use of pesticides in the past few years has weakened the frogs considerably. The rate at which they reproduce now is only  $1/4$  what it used to be and when interacting with alligators they are now twice as likely to be killed and eaten as in the past.

Write an equation describing the population of frogs under these new conditions.

$$\frac{dF}{dt} = \frac{1}{4} \times 6F - 2 \times 10AF = \frac{3}{2}F - 20AF$$

(b) Worried about the low numbers of frogs, the team of scientists decide, in addition, to introduce 10 frogs per month from another region.

Write an equation describing the population of frogs under these new conditions.

$$\frac{dF}{dt} = \frac{3}{2}F - 20AF + 120$$