Math 331.2: Homework 1 (Section 1.1–1.3)

1. Solve the following ODE's by integration or by remembering a differentiation formula.

- $y' + 2sin(2\pi x) = 0$.
- $y'' = -y$.

2. Verify that y is a solution of the given ODE.

- $y' + 5xy = 0, \quad y = ce^{-2.5x^2}$.
- $y' = y - y^2, \quad y = \frac{1}{1+ce^{-x}}$.

3. Solve the equation $y' = -1 - 2y$, $y(0) = 2$.

4. Solve the following equations

- $y^3 y' + x^3 = 0$
- $y' = e^{2x} y^2$.
- $y' = 1 + 4y^2$, $y(1) = 0$.

5. A certain population satisfies the equation $\frac{dp}{dt} = 2p - 50$.
(a) Suppose $p(0) = 15$. When will the population become extinct?
(b) Suppose $p(0) = 30$. When will the population become extinct?

6. A spherical raindrop evaporates at a rate proportional to its surface area. Write down a differential equation for $V(t)$, the volume of the raindrop as a function of time t.

7. A falling object satisfies the equation

$$\frac{dv}{dt} = 9.8 - \frac{1}{5}v\,, \quad v(0) = 0\,.$$

(a) Find the time for the object to reach 98% of its limiting velocity.
(b) At the time found in (a) how far did the object fall?

8. A falling object (of mass m=10 kg) is submitted to negligible air friction. (Take $g = 9.8m/s^2$).
(a) Write down a differential equation for the velocity of the object and solve it.
(b) An object is dropped with initial velocity 0 from a height of 200 meter. Determine how long it take for the object to reach the ground.
(c) Determine the velocity at the time of impact.

9. Radium ${}^{224}_{88}Ra$ has a half-life of 3.6 days.

1. Given 1g, how much will persist after 1 day.
2. How long does it take to have 0.001 g left?

10. A pound contains 10000 gal of water had been contaminated with $5kg$ some chemical substance. The tank is flushed out with water which contains (naturally and safely) $0.01g$ of this chemical per gallon and which flows at a rate of 300 gallons per hour. The mixture flows out at the same rate so the amount of water in the pound is constant. You should assume that water in the pound is well-mixed so that the concentration of chemical is uniform.
(a) Write down an differential equation for the amount of chemical, $S(t)$, in the pond at any given time. Take S in to be measures in grams and t in hours.
(b) Find the general solution of the equation found in (a).
(c) What is the time at which the concentration of chemical dips below $0.02g$ per gallon. How much water was needed to achieve this?