## Math 331.2: Homework 9 (Section 3.8)

1. Write the following expression as a product of trigonometric functions and graph it.

$$
\cos (\pi t)-\cos (3 t)
$$

2. For the following forced spring-mass systems determine the steady state solutions. If adequate, compute the amplitude of the steady states.
(a) $4 y^{\prime \prime}+6 y^{\prime}+2 y=-3 \cos (2 t)$.
(b) $y^{\prime \prime}+0.2 y^{\prime}+0.82 y=2 \cos (3 t)-\cos (3 t)$.
(c) $4 y^{\prime \prime}+6 y^{\prime}+2 y=e^{2 t}$.
3. Consider the spring-mass system described by the equation

$$
y^{\prime \prime}+\frac{1}{4} y^{\prime}+2 y=2 \cos (\omega t), y(0)=0, y^{\prime}(0)=2
$$

(a) Determine the steady state solution.
(b) Find the amplitude $A$ of the steady state solution in terms of $\omega$ and plot it versus $\omega$.
5.
(a) For the equation $m y^{\prime \prime}+k y=2 \cos (3 t)$ for which values of $k$ and $m$ do a resonnances occur?
(b) For the equation $16 y^{\prime \prime}+9 y=3 \cos (t)$ find the slow frequency and the fast frequency of the beat.
6. Solve the initial value problem $y^{\prime \prime}+9 y=3 \cos (\pi t), y(0)=0, y^{\prime}(0)=0$ and make a graph of the solution. Hint: Use problem 1.
7. Solve the initial value problem $y^{\prime \prime}+5 y=2 \sin (2 t), y(0)=0, y^{\prime}(0)=0$.
8. Solve the initial value problem $y^{\prime \prime}+9 y=3 \cos (3 t), y(0)=0, y^{\prime}(0)=0$ and make a graph of the solution.
9. Solve the initial value problem $y^{\prime \prime}+16 y=2 \sin (4 t), y(0)=0, y^{\prime}(0)=0$ and make a graph of the solution.

