Math 331.2: Homework 8 (Section 3.7)

1. For the following two function write them in the form $y(t) = R\cos(\omega_0 t - \delta)$ (Recall R is the *amplitude* of the oscillations, ω_0 is the *frequency* of the oscillations, δ is the *phase shift*). Graph the solutions, carefully indicating clearly on your graph R, ω_0, δ.

(a) $y(t) \;=\; 3\cos(2t) + 4sin(2t)$

(b) $y(t) \;=\; -\cos(t) + \sqrt{3}\sin(t)$

2. A mass weighing 3lb stretches a spring 3 in. Determine the corresponding mass m and spring constant k. If the spring is pushed upward a distance of 1 in and then set in motion with a downard velocity of 2 ft/sec, determine the position $y(t)$ of the mass at time t. Find the frequency, period, amplitude and phase of the motion. *Hint:* $g = 32ft/sec^2$.

3. A mass of 100 g stretches a spring 5 cm. If the mass is set in motion from its equilibrium position with a downard velocity of 10 cm/s and if there is no damping, determine the position y of the mass at time t. Find the frequency, period, amplitude and phase of the motion. *Hint:* $g = 9.8m/sec^2$.

4. A mass weighing 3lb stretches a spring 3 in. The damping constant is γ lb $\cdot$ s /ft

(a) Find the value of γ at which the system transition from damped to overdamped.

(b) Suppose $\gamma = 1$ and that the mass is set in motion from its equilibrium position with a downward velocity of 2 in/sec. Find the time at which the mass returns to its equilibrium position for the first time.

5. The position of a certain mass-spring system satisfies the initial value problem $\frac{3}{2}y'' + ky = 0$, $y(0) = 2$, $y'(0) = v$. It is observed that the period and amplitude of the motion are π and 3, respectively. Determine the spring constant k and the initial velocity v.

6. The position of a certain mass-spring system satisfies the initial value problem $y'' + y' + \frac{5}{2}y = 0$, $y(0) = 0$, $y'(0) = 2$.

(a) Is the system damped or overdamped? If it is damped determine the quasi-frequency and quasi-period.
(b) Solve the initial value problem.
(c) Plot y versus t and y' versus t.

7. The position of a certain mass-spring system satisfies the initial value problem $y'' + \frac{3}{2}y' + \frac{1}{2}y = 0$, $y(0) = 0$, $y'(0) = 2$.

(a) Is the system damped or overdamped? If it damped determine the quasi-frequency and quasi-period.
(b) Solve the initial value problem.
(c) Plot y versus t and y' versus t.

8. The position of a certain mass-spring system satisfies the initial value problem $y'' + y' + \frac{1}{4}y = 0$, $y(0) = 0$, $y'(0) = 2$.

(a) Is the system damped or overdamped? If it damped determine the quasi-frequency and quasi-period.
(b) Solve the initial value problem.
(c) Plot y versus t and y' versus t.