Math 331.2: Homework 6 (Section 3.2, 3.3, 3.4)

1. Consider the equation $t^2y'' - t(t+2)y' + (t+2)y = 0$. Show that $y_1(t) = t$ and $y_2(t) = te^t$ are two solution of the equations. Compute the Wronskian and solve the equation with initial value $y(1) = 2$, $y'(1) = 0$.

2. Compute the Wronskian of $y_1(t) = e^{\lambda t}\cos(\mu t)$ and $y_2(t) = e^{\lambda t}\sin(\mu t)$ and show that it never vanishes.
3. Use Euler formula to write the given expression in the form $a + ib$.

(a). e^{1+3i}, **(b).** e^{2+2i}/e^i, **(c).** $e^{i\pi/2}$ **(d).** $e^{i2\pi/3}$ **(e).** 2^{1-i}

For the following problems find the general solution

4. $y" - 2y' + 2y = 0$

5. $4y'' + 9y = 0$

6. $y'' - 2y' + y = 0$

7. $y" + 6y' + 13y = 0$

8. $4y'' + 17y' + 4y = 0$

For the following problems find the solution of the initial value problem, *sketch a graph of the solution* and describe its behavior for large t.

9. $y" - 2y' + 2y = 0, y(0) = 1, y'(0) = 0$

10. $4y'' + 9y = 0, y(0) = 2, y'(0) = -1$

11. $y'' - 2y' + y = 0, y(0) = 1, y'(0) = 2$

12. $y" + 6y' + 13y = 0, y(0) = 0, y'(0) = 1$

13. $4y'' + 17y' + 4y = 0, y(0) = -3, y'(0) = 2$

Hints and solutions:

1. The Wronskian is t^2e^t which is non zero if $t \neq 0$. The solution is $y(t) = 4t - 2te^{t-1}$.

2. $\lambda e^{\mu t}$.

3. (c) i, (d) $-1/2 + i\sqrt{3}/2$, (e) $2\cos(\ln(2)) - i2\sin(\ln(2))$

9. $y(t) = e^t \cos(t) - e^t \sin(t)$

10. $y(t) = 2\cos(3t/2) - 2/3\sin(3t/2)$

11. $y(t) = \frac{1}{2}e^{-3t}\sin(2t)$

12. $y(t) = -1/3e^{-4t} - 8/3e^{-t/4}$