Math 331.2: Homework 5 (Section 2.4 and Section 3.1)

In the following two problems, solve the initial value problem and determine the interval on which the solution exists

1. $y' = -4t/y\,,\ y(0) = y_0$

2. $y' + y^3 = 0\,,\ y(0) = y_0$

3. Consider the nonlinear equation $\dfrac{dy}{dt} = 2y - y^2$ Show that $v = \dfrac{1}{y}$ satisfies the linear equation $\dfrac{dv}{dt} = -2v + 1$. Solve the equation for v to find the solution for y.

For the following equations find the general solutions

4. $y'' + 2y' - 3y = 0$

5. $y'' + 5y = 0$

6. $6y'' - y' - y$

For the following equations find the solution of the given initial value problem. **Sketch the graph of the solutions**, and describe the behavior as t increases.

7. $y'' + 4y' + 3y = 0\,,\ y(0) = 2\,,\ y'(0) = -1$

8. $y'' + 3y = 0\,,\ y(0) = -2\,,\ y'(0) = 3$

9. Find the solution of the initial value problem $2y'' - 3y' + y = 0\,,\ y(0) = 2, y'(0) = \dfrac{1}{2}$. Sketch a graph of the solution. Determine the maximum value of the solution. Find the point where the solution is 0.

10. Consider the equation $t^2y'' - 2ty - 4y = 0$. Find the general solution of this equation by trying function of the form t^r some r.

11. Consider $4y'' - y = 0$, $y(0) = 2$. Find the value of the initial $y'(0) = \beta$ such that $\lim_{t\to\infty} y(t) = 0$.

Hints and solutions:

1. $y(t) = \pm\sqrt{y_0^2 - 4t^2}$ the solution exists if $-|y_0|/2 \le t \le |y_0|/2$

2. $y(t) = \dfrac{y_0}{\sqrt{2ty_0^2 + 1}}$, the solution exists if $-\dfrac{1}{2y_0^2} < t < \infty$

4. $y(t) = c_1 e^t + c_2 e^{-3t}$

5. $y(t) = c_1 + c_2 e^{-5t}$

6. $y(t) = c_1 e^{-t/3} + c_2 e^{t/2}$

7. $y(t) = \dfrac{5}{2} e^{-t} - \dfrac{1}{2} e^{-3t}$

8. $y(t) = -1 - e^{-3t}$

9. The maximum is at $t = \ln(9/4)$ and the zero is at $t \ln(9)$.

10. $y(t) = c_1 t^4 + c_2 t^{-1}$

11. $\beta = -1$.