Math 331.2: Homework 3 (Section 2.3)

1. After charging $5000 to his credit card which carries a yearly interest of 20 %, Bob decides to stop using his card and to pay back $100 every month. Determine how long Bob will need to pay off his debt and how much interest will he end up paying?

2. Jane would like to buy a condo and decides she can afford a monthly payment of $1500. How much money can she borrow from the bank if you assume that she is taking a 30 year loan with a fixed interest rate of 6 %? What if the rate is 5 % instead?

3. Bob would like to retire at age 55 and imagine he might live another 30 years after retiring. His retirement money is an account which earns a fixed interest rate of 5%.

(a) How much money should he save before retiring if plans to need $50'000 per year after he retires?

(b) After some thought, Bob decides to take inflation into account. He assumes that the rate of inflation after he retires will be a constant 2% and he wants to keep his lifestyle unchanged. How much money will he need t years after his retirement?

(c) Now if he takes into account inflation as found (b) how much money should he save before retiring?

4. A tank contains 100 gal of pure water in which 5 lb of sugar is dissolved. At time 0 a sugar-water solution containing 0.2 lb of sugar per gallon enters the tank at a rate of 3 gallons per minute. At the same time a drain at the bottom of the tank is opened and the sugar solution leaves the tank at a rate of 3 gallons per minute. Assume that the solution in the tank is perfectly mixed.

(a) What is the sugar content of the water after 20 minutes?

(b) How long will it take for the sugar content in the tank to reach 15 lbs?

(c) What will be the eventual sugar content of the tank?

5. After a drought a water reservoir of capacity 200 liters is only half-full with pure water. The rate at which water is taken from the reservoir is 1 liters per hour. To replenish the reservoir it is decided to pump water into the half empty reservoir at a rate of 2 liters per hour until the reservoir is full. The water pumped into the reservoir is mistakenly polluted with salt with a concentration of 25g per liter. Write down the equation for the amount of salt in the pound at time t. How much salt will there be in the reservoir when the reservoir is full?

6. *Newton's law of cooling:* Suppose the ambient temperature is known and given by a function $T(t)$. If an object with temperature $u(t)$ is in an environment at temperature $T(t)$, then Newton's law of cooling tells us the temperature of the object satisfies the differential equation

$$\frac{du}{dt} = -k[u - T(t)]$$

where k is a positive constant which depends only on the nature of the object.

(a). The dead body of a man is found at 10 pm in a room at temperature $72^o F$. The body temperature is $80^o F$ and it is estimated that the rate of cooling k is 10 if t is measured in days. Determine the time of death. ($98.2^0 F$ is the normal body temperature).

(b). A can of beer at $40^o F$ is placed into a room at temperature $70^o F$. After 20 minutes the temperature of the beer is $55^o F$. What is the temperature of the beer after 30 minutes.

Hints and solutions:
1 8.95 years and $5750.56.

2 For 6% it is $250410 and for 5% it is 279673.

3 (a) At least $776,870. (b) $50000e^{t/50}$. (c) At least $ 2344259.

5 3750 g of salt.

6 (a) About 2 hours and 50 minutes. (b) $59.4^o F$