Math 331.2: Homework 2 (Section 2.1 and 2.2)

1. Classify the following equations as *linear* or *nonlinear* and *separable* or not. (Do not solve them.)

(a) $\dfrac{dy}{dt} = \dfrac{ty - (t+3)y}{y^2}$.

(b) $y\dfrac{dy}{dt} + ty = cos(t)y^2$.

(c) $\dfrac{dy}{dt} + 2ty = \cos(t + 2y)$.

(d) $\dfrac{dy}{dt} = y^2(t + \cos(t))$.

2. Find the general solution for $y' + 3y = t + e^{-2t}$. Describe the behavior of the solutions as $t \to \infty$.

3. Find the general solution for $y' - 2y = t^2e^{2t}$. Describe the behavior of the solutions as $t \to \infty$.

4. Find the general solution for $ty' + 2y = \sin(t)$.

5. Solve the initial value problem $y' - y = e^{2t}$, $y(0) = 3$.

6. Solve the initial value problem $t^3y' + 3t^2y = e^{-t}$, $y(-1) = 0$.

7. Find the general solution of $y' = x^2/y$ and then solve the initial value problem for the initial condition $y(1) = -1$.

8. Solve the differential equation $y' = \dfrac{x^2}{y^2(1 + x^3)}$.

9. Solve the differential equation $y' = \dfrac{3x^2 - 1}{3 + 2y}$.

10. Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x - e^{-x}}{y + e^y}$.

11. Solve the initial value problem $y' = (1 - 2t)y^2$, $y(0) = -2$.

12. Solve the initial value problem $\sin(2t) + \cos(3y)\frac{dy}{dt}$, $y(\pi/2) = \pi/3$.

Solutions:

2 $y(t) = Ce^{-3t} + (t/3) - 1/9 + e^{-2t}$. For large t, $y(t)$ is asymptotic to the line $t/3 - 1/9$.

3 $y(t) = Ce^{2t} + \frac{t^3}{3}e^2 t/3$, $y(t)$ diverges as $t \to \infty$.

4 $y(t) = \frac{1}{t^2}(c - t\cos(t) + \sin(t))$

5 $y(t) = e^{2t} + 2e^t$.

6 $y(t) = \frac{1}{t^3}(e - e^{-t})$.

7 $y(x) = \pm\sqrt{\frac{x^3}{3} + C}$ and $y(t) = -\sqrt{\frac{x^3}{3} + 2/3}$

8 $y(x) = (\ln(1 + x^3) + C)^{1/3}$

9 In implicit form $y^2 + 3y = x^3 - x + C$ or in explicit form $y = -\frac{3}{2} \pm \frac{1}{2}\sqrt{9 + 4(x^3 - x + C)}$

10 In implicit form $\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x}$.

11 $y(t) = \dfrac{1}{t^2 - t - \frac{1}{2}}$

12 $y(t) = \frac{1}{3}\arcsin\left(\frac{3}{2}\cos(2t) + \frac{3}{2}\right)$.